Wealth Dynamics and the Persistence of Specialization*

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Abstract

This paper shows how financial frictions can make sectoral specialization persistent, even when sectors have identical fundamentals. I develop a dynamic multisector general equilibrium model with heterogeneous entrepreneurs who face collateral constraints and persistent productivity shocks. Because borrowing capacity depends on wealth, the allocation of wealth across productive and unproductive entrepreneurs determines effective sectoral productivity and future accumulation. The key result is that the persistence of these distortions depends on the price regime. In a closed economy, relative prices respond to sectoral scarcity: initially disadvantaged sectors become more profitable, which accelerates wealth accumulation among constrained productive firms and promotes convergence. In a small open economy, relative prices are fixed at world levels, so this corrective force is absent. Initial differences in the wealth-productivity distribution therefore generate persistent differences in sectoral size and specialization, even with identical technologies and preferences. Trade integration does not eliminate misallocation; it changes how misallocation propagates through wealth accumulation. Firm- and sector-level evidence from Peru is consistent with two implications of the mechanism: persistent productivity dispersion within sectors and gradual sectoral responses to external demand shocks.

Keywords: Wealth Distribution, Sectoral Heterogeneity, Persistence, Misallocation
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Preliminary and frequently updated draft. Comments welcome.

Link to the latest version

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1 Introduction

> "Peru needs products from advanced countries, not experiments with Peruvian products."
>
> *President Augusto B. Leguía, to the Peruvian entrepreneur Juan Alberto Grieve.*[1]

While some economies have experienced substantial structural transformation, the productive structure of many economies remains remarkably persistent over long periods of time. A natural interpretation is that these economies are simply specializing according to their comparative advantage. However, many of them also exhibit high levels of labor and capital misallocation, slowly growing firms, and significant financial and operational frictions. If productive sectors coexist with constrained firms and persistent distortions, it is natural to ask whether observed specialization patterns reflect efficient allocation or instead arise from frictions that slow the reallocation of capital across activities. If so, financial frictions may not only slow structural transformation in the short run, but also shape the long-run evolution of sectoral specialization through their interaction with equilibrium prices and wealth accumulation. **This paper** studies this mechanism through a dynamic multisector framework in which financial frictions and the distribution of wealth jointly determine sectoral allocation and specialization dynamics.

Consider an economy in which entrepreneurs differ in productivity but face financial constraints when operating or expanding firms. With frictionless financial markets, capital would flow toward the most productive entrepreneurs within each sector, allowing productive activities to expand regardless of the initial distribution of wealth. However, if access to financing is limited by entrepreneurs' wealth, productive but constrained firms may be unable to scale up, slowing the reallocation of capital toward high-productivity activities.

These distortions can also shape specialization dynamics across sectors. Even when sectors share identical technologies and fundamentals, sectors with a more favorable allocation of wealth toward productive entrepreneurs will initially produce more and accumulate wealth faster. In a closed economy, however, these asymmetries generate relative price adjustments: sectors with lower effective supply become more profitable, accelerating wealth accumulation among constrained but highly-productive firms. As a result, initial differences would tend to dissipate over time. In contrast, in open economies, domestic relative scarcity no longer translates into higher sectoral prices, as relative prices will be tied to global market conditions. In particular, in a small open economy that takes world prices as given, this adjustment mechanism is weakened. Initial allocation

[1]Juan Alberto Grieve (1878–1950) was a Peruvian inventor who designed and built the first internal-combustion engine in South America and one of the first automobiles in the region. His vehicles, adapted to rough local conditions at the time, were reportedly priced around half the cost of importing comparable European models of similar power. Despite this, his project failed to scale, in part due to limited access to financing and weak institutional support. This episode reflects a broader environment in which domestic industrial experimentation faced skepticism and limited support, potentially constraining the reallocation of resources toward more productive local activities. Such patterns are consistent with settings where financial frictions and coordination failures hold back the expansion of efficient firms in constrained sectors. See https://www.britannica.com/biography/Juan-Alberto-Grieve.

advantages are therefore no longer counteracted by relative price adjustment, allowing distortions to translate into persistent specialization patterns.

I explore these mechanisms building on the heterogeneous-agent framework with financial frictions in Moll (2014), extending it to a dynamic multisector general equilibrium environment with endogenous relative prices and sectoral reallocation. Goods produced in different sectors are combined through a constant elasticity of substitution (CES) aggregator and used for consumption and investment. Entrepreneurs operate technologies in a given sector and face collateral constraints and idiosyncratic productivity risk. Since access to capital depends on wealth, wealth accumulation shapes firm expansion and the allocation of resources both within and across sectors over time. I study equilibrium dynamics in two regimes: a closed economy with endogenous relative prices and a small open economy that takes world prices as given under financial autarky, allowing sectoral trade imbalances while maintaining aggregate trade balance. Comparing these environments allows the analysis to isolate the role of price adjustment in shaping the evolution of sectoral allocation.[2]

The model delivers two main theoretical results. First, in a closed economy, endogenous relative prices act as a stabilizing force. Sectors with lower effective supply become relatively more profitable, raising the returns to productive but financially constrained entrepreneurs and accelerating wealth accumulation in initially disadvantaged sectors. As a result, differences in sectoral allocation and productivity tend to dissipate over time.

Second, this adjustment mechanism weakens substantially in open economies. When relative prices are tied to world market conditions, domestic scarcity no longer generates compensating price responses. In a small open economy with exogenous world prices, sectors with an initial allocation advantage accumulate wealth more rapidly and expand persistently, even when sectors share identical technologies and fundamentals. The model therefore generates persistent differences in specialization patterns arising solely from initial differences in the distribution of wealth and the allocation of capital.

The contribution is to show that the self-correction logic in one-sector models with financial frictions changes once production is multisector and prices differ across sectors. In a one-sector economy, wealth dynamics affect aggregate productivity through selection. In a multisector economy, the same wealth dynamics also shape sectoral composition. Whether this generates convergence or persistence depends on the price regime: endogenous domestic prices compress initial asymmetries, while fixed world prices allow them to persist.

Quantitatively, the model implies that relatively small initial distortions can generate large and persistent differences in sectoral composition over long horizons. Because wealth accumulation is gradual, the effects of financial frictions propagate dynamically through firms' balance sheets and

[2] Appendix J presents a stripped-down analytical benchmark showing how temporary productivity differences can generate persistent specialization patterns in a small open economy with fixed world prices and sector-specific capital accumulation.

sectoral profitability. Economies with similar aggregate resources and technologies but different initial allocations of wealth can therefore follow substantially different specialization paths. The model also predicts that the speed of convergence depends critically on the degree of financial frictions and on the extent to which domestic prices adjust to sectoral scarcity. In open economies, persistent relative price differences at the world level amplify these dynamics, slowing reallocation and reinforcing initial allocation advantages.

Peru provides a useful environment to study these mechanisms. As Figure 1 shows, despite periods of growth, trade integration, and large changes in external demand conditions, production and exports have remained concentrated in a relatively persistent set of sectors over the last few decades. At the same time, firm-level data reveal substantial productivity dispersion and heterogeneous expansion dynamics across firms and sectors.

Motivated by these patterns, I use firm-level and sector-level evidence from Peru to evaluate key mechanisms emphasized by the model. Using firm-level panel data *Encuesta Económica Anual* (EEA) (Instituto Nacional de Estadística e Informática (INEI), 2001-2019), the empirical analysis shows that more productive firms tend to expand capital and labor more strongly following favorable external demand shocks, consistent with heterogeneous accumulation dynamics under financial frictions. In addition, sector-level evidence using household survey data (Instituto Nacional de Estadística e Informática (INEI), 2004-2023) shows that sectors exposed to favorable external demand conditions experience persistent increases in employment shares over subsequent years. Together, these patterns are consistent with the idea that external shocks affect long-run specialization through endogenous accumulation and gradual reallocation dynamics.[3]

Related Literature. The paper relates to four strands of literature. First, it relates to work studying the interaction between financial frictions, wealth heterogeneity, and aggregate productivity, including Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Midrigan and Xu (2014), Buera, Kaboski, and Shin (2011), and Moll (2014). This literature emphasizes how distortions to capital allocation affect firm growth and aggregate outcomes. The present paper extends these mechanisms to a multisector environment in which wealth distributions shape sectoral allocation and specialization dynamics over time.

Second, the paper connects to the literature on trade and financial frictions. Antràs and Caballero (2009) study how financial development affects trade patterns, while Manova (2008) shows how credit constraints shape export activity. Relative to this literature, the focus here is not on trade flows themselves, but on how wealth distributions and financial frictions interact with prices to generate persistent differences in sectoral allocation across otherwise similar economies. This extension makes relative price adjustment central for determining whether initial distortions are

[3] Appendix A presents complementary cross-country evidence on the persistence of sectoral specialization patterns using international trade data harmonized across countries and sectors. The appendix documents that specialization patterns remain highly persistent over long horizons despite substantial changes in global trade integration and external demand conditions.

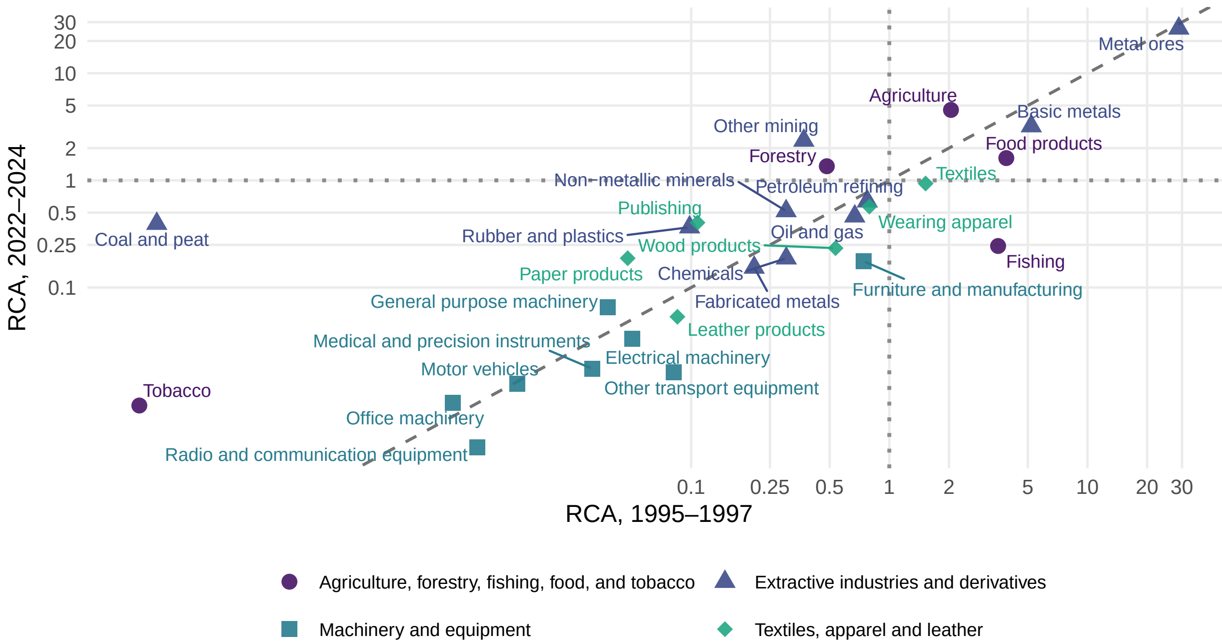


Figure 1: Sectoral Revealed Comparative Advantage in Peru, 1995–1997 vs. 2022–2024

Notes: The figure plots revealed comparative advantage (RCA) for Peruvian exports across ISIC Rev. 3 two-digit sectors in two periods: 1995–1997 (horizontal axis) and 2022–2024 (vertical axis). RCA is defined as $RCA_{i,t} = (X_{Peru,i,t}/X_{Peru,t}) / (X_{World,i,t}/X_{World,t})$ where $X_{\text{Peru},i,t}$ denotes Peruvian exports in sector i at time t, $X_{\text{Peru},t}$ total Peruvian exports, and $X_{\text{World},i,t}$ and $X_{\text{World},t}$ the corresponding world aggregates. Both axes are reported on a logarithmic scale. The dashed vertical and horizontal lines mark $RCA = 1$, the threshold for comparative advantage. The dashed 45-degree line indicates unchanged specialization between periods; sectors above (below) the line increased (decreased) their revealed comparative advantage over time. Colors and marker shapes identify broad industry groups. Trade data are from the BACI International Trade Database harmonized to ISIC Rev. 3 classifications.

amplified or reduced over time.

Third, the paper relates to work on reallocation and adjustment dynamics, such as Caliendo, Dvorkin, and Parro (2019) and Dix-Carneiro and Kovak (2017), which study how frictions slow the movement of factors across activities following shocks. In contrast, the mechanism emphasized here operates through within-sector wealth accumulation and endogenous price responses, generating persistence even in the absence of barriers to sectoral mobility.

Fourth, the paper connects to the literature on firm dynamics and selection, including Hopenhayn (1992), Melitz (2003), and Luttmer (2007). In these environments, selection operates through productivity differences. Here, selection is mediated by wealth and financial constraints, so productive but constrained firms may be unable to expand, slowing reallocation toward high-productivity producers.

More generally, the paper contributes to these literatures by showing how wealth distributions and financial frictions interact with price adjustment to shape sectoral allocation and persistence. While Ventura (1997) and Acemoglu and Ventura (2002) emphasize how general equilibrium price adjustments can offset initial differences across economies, the mechanism here highlights how financial frictions can instead propagate initial distortions through endogenous wealth accumulation.

Outline of paper. The rest of the paper proceeds as follows. Section 2 develops a multisector model with heterogeneous entrepreneurs facing financial frictions, in which wealth distributions determine production, prices, and accumulation. Section 3 studies a closed economy, showing how endogenous price adjustment reduces initial asymmetries and promotes convergence through reallocation and selection. Section 4 introduces a small open economy with fixed goods prices, isolating the role of price adjustment and showing how initial differences in wealth and allocation translate into persistent differences in sectoral composition. Section 5 provides firm-level evidence consistent with the mechanism, and Section 6 concludes.

2 A Multisector Model of Wealth Dynamics and Specialization

This section presents a multisector extension of the heterogeneous-agent model with financial frictions developed by Moll (2014). The key difference is that production takes place in multiple sectors whose outputs are combined through a CES demand system, so that relative prices and specialization patterns emerge endogenously in general equilibrium. To characterize how wealth heterogeneity shapes sectoral allocation, the model introduces sector-specific wealth distribution objects that track both the joint distribution of wealth and productivity within sectors and the allocation of aggregate wealth across sectors. These objects make it possible to characterize how sectoral productivity, prices, and specialization patterns evolve endogenously over time.

The environment described in this section focuses on a closed-economy benchmark that isolates the core mechanism of the paper. Sectors in this setting can be interpreted broadly as industries, regions, or economic activities. More explicit interpretations of sectors will be introduced in the next section, which develops open-economy extensions of the model.

2.1 Setup, Agents, and Final Consumption

I consider an economy with many sectors indexed by $i \in I$. Time is continuous and denoted by t. There are two types of infinitely lived agents: workers (w) and entrepreneurs (e). The total mass of workers is fixed at L. Each worker is endowed with one efficiency unit of labor that they supply inelastically but can allocate across sectors. Workers do not have access to a savings technology, so they live "hand-to-mouth", as in Kaplan and Violante (2014).

Entrepreneurs are attached to a given sector, in which they operate a production technology using capital and labor. In addition, entrepreneurs make forward-looking decisions over consumption and savings based on their stock of financial wealth a. Financial wealth can be transformed into productive capital k, used in the entrepreneur's sector of operation, or consumed.

The difference in mobility between workers and entrepreneurs is similar to the assumption used in Kleinman, Liu, and Redding (2023), where capitalists in a spatial economy are attached to their regions. This restriction helps incorporate forward-looking behavior while keeping the state space tractable. In particular, in the present model the state of the economy at any point in time t can be characterized using $g_i(a, z; t)$, the joint densities over productivity z and wealth a for entrepreneurs operating in each sector i.

I assume that all sectoral output $Y_i(t)$ is transformed into a final composite good through a constant elasticity of substitution (CES) aggregator:

$$Y(t) = \left(\sum_{i \in I} \upsilon_i^{\frac{1}{\eta}} Y_i(t)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{1}$$

where η denotes the elasticity of substitution across sectors and υ_i are sector-specific weights. This implies the standard CES demand system:

$$D_i(t) = \upsilon_i \left(\frac{p_i(t)}{P(t)} \right)^{-\eta} D(t), \qquad P(t)^{1-\eta} = \sum_{i \in I} \upsilon_i p_i(t)^{1-\eta}. \tag{2}$$

Here, $D_i(t)$ denotes sectoral demand and $D(t)$ denotes total absorption, which includes consumption by both types of agents, the replacement of depreciated capital, and the accumulation of financial wealth.

Workers supply their labor efficiency units to entrepreneurs in a competitive labor market at wage $w(t)$. For each hand-to-mouth worker, consumption expenditure is determined by their labor income, so that $P(t)c_w(t) = w(t)L$. For entrepreneurs, consumption expenditure is determined by an intertemporal optimization problem that depends on prices, productivity, and their stock of financial wealth, and jointly determines their savings behavior.

In equilibrium, sectoral output is absorbed through sectoral demands $\{D_i(t)\}_{i \in I}$, which correspond to the allocation of the final good across sectors. While $D(t)$ summarizes total absorption, it will be useful to distinguish between the portion allocated to current consumption and the portion used for capital replacement and the accumulation of financial wealth. This decomposition will be made explicit in the equilibrium section.

2.2 Production, Technology, and Budgets

Each entrepreneur has access to a technology of production that allows them to use k units of capital and ℓ units of labor to produce

$$y_i = (zk)^{\alpha_i} \ell^{1-\alpha_i} \tag{3}$$

units of output of the good in industry i, where $\alpha_i \in (0,1)$. Capital depreciates at the same rate δ in every sector. Productivity z follows a continuous-time, sector-specific ergodic Markov process:

$$dz = \mu_i(z)\, dt + \sigma_i(z)\, dW, \tag{4}$$

where $\mu_i(z)$ and $\sigma_i(z)$ denote the drift and diffusion functions, respectively, and W is a standard Brownian motion. I assume that this process admits a stationary distribution in each sector, ensuring that productivity remains bounded in distribution over time.

In addition to hiring workers at wages $w(t)$ in a competitive labor market, they can rent capital from other entrepreneurs in a competitive capital rental market at a rental rate $R(t)$, which is equal to the user cost of capital $R(t) = r(t) + \delta$. So, for an entrepreneur in sector i, current productivity z, and wealth $a(t)$, wealth evolves according to:

$$\dot{a} = p_i(zk)^{\alpha_i}\ell^{1-\alpha_i} - w\ell - (r+\delta)k + ra - Pc \tag{5}$$

Entrepreneurs face financial constraints that limit their ability to transform wealth into productive capital. In particular, capital choices are subject to a collateral constraint of the form

$$k \leq \lambda_i a, \tag{6}$$

where $\lambda_i \geq 1$ denotes the tightness of financial constraints in sector i. A higher λ_i corresponds to greater pledgeability of assets and more developed financial intermediation.

This constraint implies that entrepreneurs cannot freely arbitrage between the marginal product of capital and its user cost. In particular, when the constraint binds, capital is proportional to internal wealth, so that more productive but less wealthy entrepreneurs may operate below their efficient scale.

As a result, the allocation of capital depends on the joint distribution of productivity and wealth within each sector. This feature will be central for linking wealth dynamics to sectoral productivity and specialization in general equilibrium.

2.3 Entrepreneur's Problem

Entrepreneurs choose consumption and savings dynamically, taking prices $\{p_i(t), w(t), r(t)\}$ as given. Their decisions determine the evolution of wealth and, through the collateral constraint, their future production capacity.

Let $V_i(a, z; t)$ denote the value function of an entrepreneur in sector i with wealth a and productivity z at time t. The entrepreneur solves

$$V_i(a, z; t) = \max_{\{c(s), k(s), \ell(s)\}_{s \geq t}} \mathbb{E}\left[\int_t^{\infty} e^{-\rho(s-t)} u(c(s))\, ds\right], \tag{7}$$

subject to the law of motion for wealth, Equation 9; the collateral constraint, Equation 6, and the stochastic process for productivity, Equation 4. Throughout, preferences are logarithmic, so that $u(c) = \ln c$.

The entrepreneur's problem can be decomposed into a static and a dynamic component. The static component determines optimal input choices (k, ℓ) at each point in time, given wealth a, productivity z, and prices, and is summarized by the profit function $\Pi_i(a, z)$ defined in Equation 8. The dynamic component consists of choosing consumption and savings over time, taking as given the profit function and the stochastic evolution of productivity.

This separation is possible because input choices (k, ℓ) affect the entrepreneur's problem only through current-period profits and do not enter the law of motion for wealth except through their impact on current income. Given prices and states (a, z), the static problem determines the maximal flow payoff $\Pi_i(a, z)$ independently of intertemporal considerations. The dynamic problem then consists of allocating this flow of resources over time through consumption and savings decisions. As a result, solving the static and dynamic problems sequentially yields the same solution as the joint problem.

Static problem. It is convenient to define the entrepreneur's static profit function as the solution to the within-period problem:

$$\Pi_i(a, z) = \max_{k,\ell} \left\{ p_i (zk)^{\alpha_i} \ell^{1-\alpha_i} - w\ell - (r + \delta)k \text{ s.t. } k \leq \lambda_i a \right\}. \tag{8}$$

This formulation summarizes the effect of prices, productivity, and financial constraints on current income. Using this definition, the law of motion for wealth can be written as

$$\dot{a} = \Pi_i(a, z) + ra - Pc. \tag{9}$$

Lemma 1 (Static allocation). *Given prices $\{p_i, w, r\}_{i \in I}$, the solution to the entrepreneurs' static problem implies that capital demand, labor demand, and profits are given by*

$$k_i(a, z) = \begin{cases} \lambda_i a, & \text{if } z \geq \underline{z}_i, \\ 0, & \text{otherwise}, \end{cases}$$

$$\ell_i(a, z) = \left(\frac{1 - \alpha_i}{w/p_i} \right)^{\frac{1}{\alpha_i}} z \, k_i(a, z),$$

$$\Pi_i(a, z) = \lambda_i a \cdot \max \left\{ \pi_i z - (r + \delta), \, 0 \right\},$$

where

$$\pi_i = p_i \alpha_i \left(\frac{1 - \alpha_i}{w/p_i} \right)^{\frac{1-\alpha_i}{\alpha_i}},$$

and the productivity cutoff $\underline{z}_i$ satisfies

$$\pi_i \underline{z}_i = r + \delta.$$

Proofs in Appendix B.

A key implication of the static solution is that optimal policies are linear in wealth. Because both production and financial constraints scale proportionally with assets, wealth affects decisions only through a scaling factor, while productivity determines the intensive margins of behavior. This property will be crucial for aggregation, as it allows the distribution of wealth across productivity levels to summarize the state of the economy and its evolution over time.

Dynamic problem. Using the static allocation results, the entrepreneur's problem reduces to a consumption-savings problem with a stochastic return on wealth. In particular, the law of motion for wealth can be written as

$$\dot{a}(t) = \Gamma_i(z,t)\, a(t) - P(t)c(t), \tag{10}$$

where

$$\Gamma_i(z,t) \equiv \lambda_i \max\left\{\pi_i(t)z - (r(t) + \delta),\, 0\right\} + r(t) \tag{11}$$

denotes the total return on wealth, combining profits from production and interest income.

The entrepreneur's dynamic problem is therefore

$$\max_{\{c(s)\}_{s \geq t}} \mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)} \ln(c(s))\, ds\right] \quad \text{s.t.} \quad \dot{a}(s) = \Gamma_i(z,s)\, a(s) - P(s)c(s), \tag{12}$$

and can be represented by a Hamilton-Jacobi-Bellman (HJB) equation provided in Appendix C. Under logarithmic preferences, the solution of the problem implies that consumption expenditure is proportional to wealth:

$$P(t)c(t) = \rho a(t), \tag{13}$$

Lemma 2 (Wealth accumulation)**.** *Under logarithmic preferences, individual wealth evolves according to*

$$\dot{a}(t) = s_i(z,t)a(t), \tag{14}$$

where

$$s_i(z,t) \equiv \Gamma_i(z,t) - \rho$$

denotes the net saving rate of entrepreneurs with productivity z in sector i. Proof in Appendix C.

Differences in productivity therefore translate directly into heterogeneous wealth accumulation, governing the evolution of the wealth distribution over time.

2.4 Wealth Distributions and Aggregation

Because optimal policies and wealth accumulation are linear in assets, aggregate outcomes depend only on the distribution of wealth across productivity levels and sectors.

Let $g_i(a, z; t)$ denote the joint density of entrepreneurs with wealth a and productivity z in sector i at time t. Aggregate wealth in the economy and in sector i are given by

$$A(t) = \sum_{i \in I} A_i(t), \qquad A_i(t) = \int a\, g_i(a, z; t)\, da\, dz. \tag{15}$$

Definition 3 (Wealth distributions)**.** Relative to aggregate wealth quantities $A(t)$ and $A_i(t)$, define the following wealth-weighted distributions:

- The share of aggregate wealth held in sector i:

$$\mathcal{O}_i(t) = \frac{A_i(t)}{A(t)}.$$

- The within-sector wealth-weighted productivity density:

$$o_i(z, t) = \frac{1}{A_i(t)} \int a\, g_i(a, z; t)\, da.$$

- The sector-productivity joint wealth density:

$$\omega_i(z, t) = \frac{1}{A(t)} \int a\, g_i(a, z; t)\, da = \mathcal{O}_i(t)\, o_i(z, t).$$

- The economy-wide wealth density

$$\omega(z, t) = \sum_{i \in I} \omega_i(z, t).$$

These objects cumulative wealth share distributions within sectors $\Omega_i(z, t) \equiv \int_0^z o_i(x, t)\, dx$, and for the economy as a whole $\Omega(z, t) \equiv \sum_{i \in I} \mathcal{O}_i(t) \cdot \Omega_i(z, t) = \sum_{i \in I} \int_0^z \omega_i(x, t)\, dx$.

The objects $\mathcal{O}_i(t)$ and $o_i(z, t)$ summarize, respectively, the distribution of wealth across sectors and across productivity levels within sectors. Their product, $\omega_i(z, t)$, characterizes the joint distribution of wealth across sectors and productivity levels and is sufficient to determine aggregate allocations and production.

Lemma 4 (Aggregation)**.** *Because optimal policies are linear in wealth, aggregate allocations depend only on the wealth-weighted distribution over productivity. In particular, for any allocation or profit variable*

$x_i(a,z)$ that is linear in wealth,

$$\int x_i(a,z)\, g_i(a,z;t)\, da\, dz = A(t) \int x_i(z)\, \omega_i(z,t)\, dz. \tag{16}$$

Hence, the joint wealth density $\omega_i(z,t)$ is sufficient to characterize aggregate allocations and production.

2.5 Closed Economy Equilibrium

Definition 5 (Closed economy equilibrium)**.** Given initial wealth distributions $\{\omega_i(z,0)\}_{i\in I}$, wealth levels $\{A_i(0)\}_{i\in I}$, and labor endowment $\bar{L}$, a competitive closed economy equilibrium is a path of prices

$$\{w(t), r(t), p_i(t)\}_{i\in I, t\geq 0},$$

allocation rules

$$\{k_i(z,t), \ell_i(z,t), c_i(z,t)\}_{i\in I, t\geq 0},$$

and wealth distributions

$$\{\omega_i(z,t)\}_{i\in I, t\geq 0},$$

such that:

1. Entrepreneurs optimize given prices.
2. Factor markets clear:

$$\bar{L} = A(t) \sum_{i\in I} \int \ell_i(z,t)\, \omega_i(z,t)\, dz,$$
$$A(t) = A(t) \sum_{i\in I} \int k_i(z,t)\, \omega_i(z,t)\, dz.$$

3. Goods markets clear:

$$Y_i(t) = D_i(t), \qquad \forall i \in I,$$

   where sectoral output is

$$Y_i(t) = A(t) \int y_i(z,t)\, \omega_i(z,t)\, dz.$$

4. Sectoral demands satisfy the CES system

$$D_i(t) = \upsilon_i \left(\frac{p_i(t)}{P(t)}\right)^{-\eta} D(t),$$

   with price index

$$P(t)^{1-\eta} = \sum_{i\in I} \upsilon_i p_i(t)^{1-\eta}.$$

5. Aggregate absorption satisfies

$$P(t)D(t) = P(t)C(t) + \dot{A}(t) + \delta A(t),$$

where aggregate consumption expenditure is

$$P(t)C(t) = w(t)\bar{L} + \rho A(t).$$

6. Wealth distributions evolve consistently with individual wealth accumulation and the productivity process.

2.6 Aggregation and Dynamics

The linearity of optimal policies in wealth implies that aggregate dynamics are governed by the distribution of wealth across entrepreneurs. In equilibrium, sectoral productivity reflects endogenous selection, while wealth accumulation depends jointly on profitability, prices, and the allocation of wealth across sectors. The following proposition summarizes these aggregation and dynamic relationships.

Proposition 6 (Wealth distributions, sectoral productivity, and accumulation)**.** *Because individual policies are linear in wealth, sectoral allocations and productivity depend on the wealth-weighted productivity distribution. Sectoral output can be written as*

$$Y_i(t) = Z_i(t)K_i(t)^{\alpha_i}L_i(t)^{1-\alpha_i},$$

where

$$Z_i(t) = \left(E_{o_i,t}[z \mid z \geq \underline{z}_i(t)]\right)^{\alpha_i}.$$

Sectoral wealth evolves according to

$$\dot{A}_i(t) = \alpha_i p_i(t)Y_i(t) + r(t)\big[A_i(t) - K_i(t)\big] - \delta K_i(t) - \rho A_i(t).$$

Thus, the distribution of wealth across productivity states determines sectoral productivity, profitability, and future accumulation.

Proof and factor-price conditions in Appendix D.

The proposition highlights the channel linking wealth distributions to specialization dynamics. Sectoral productivity depends on how wealth is allocated across productivity types within each sector, while sectoral wealth accumulation depends on profits, prices, and capital use. Relative to the one-sector environment in Moll (2014), sectoral wealth and sectoral capital need not coincide: a sector can use more capital than the wealth held by its entrepreneurs, or it can hold more wealth than the capital used in production. As a result, the common interest rate affects sectoral wealth accumulation through a net asset position channel. An increase in the interest rate raises wealth accumulation in sectors that are net lenders, with $A_i(t) > K_i(t)$, and lowers it in sectors that are net borrowers, with $A_i(t) < K_i(t)$. Equilibrium prices and the interest rate therefore jointly govern the speed at which wealth reallocates across sectors over time, and this reallocation changes future sectoral productivity and specialization.

3 Closed Economy Dynamics

The dynamics of the closed economy are governed by a feedback loop between wealth, production, and prices. Firms with higher productivity accumulate wealth faster, which allows them to operate at larger scale and increases their contribution to sectoral output. As resources are reallocated toward more productive firms, sectoral productivity rises, affecting relative prices and factor returns in general equilibrium. These price adjustments, in turn, influence savings decisions and the allocation of wealth across sectors. In the closed economy, this feedback tends to reduce initial asymmetries, as changes in relative prices partially offset the expansion of sectors that experience faster productivity growth.

3.1 Evolution of Wealth Shares

Given the stochastic evolution of productivity and the optimal saving policy, the individual state (z, a) evolves according to

$$\begin{aligned} dz &= \mu_i(z)\,dt + \sigma_i(z)\,dW, \\ da &= s_i(z,t)\,a\,dt, \end{aligned} \tag{17}$$

where $s_i(z,t) \equiv \Gamma_i(z,t) - \rho$ denotes the net growth rate of wealth implied by optimal behavior.

These dynamics induce a law of motion for the joint distribution $g_i(a, z; t)$ via the associated Kolmogorov Forward equation. As shown in Appendix G, applying this equation to the joint process and exploiting the linearity of policies in wealth implies that the wealth-weighted distribution $\omega_i(z,t)$ satisfies

$$\frac{\partial \omega_i(z,t)}{\partial t} = \left[s_i(z,t) - \frac{\dot{A}(t)}{A(t)} \right] \omega_i(z,t) - \frac{\partial}{\partial z}\left[\mu_i(z)\omega_i(z,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial z^2}\left[\sigma_i^2(z)\omega_i(z,t)\right]. \tag{18}$$

The first term captures differential wealth accumulation across productivity levels, relative to aggregate wealth growth. The remaining terms reflect the stochastic evolution of productivity.

This law of motion is the point at which the model departs most directly from the one-sector environment in Moll (2014). In Moll's framework, wealth accumulation changes the economy-wide wealth-weighted productivity distribution. Here, the same force operates within each sector, but it also governs the evolution of sectoral wealth shares. In particular, the within-sector distribution evolves according to

$$\frac{\partial o_i(z,t)}{\partial t} = \left[s_i(z,t) - \bar{s}_i(t)\right] o_i(z,t) - \frac{\partial}{\partial z}\left[\mu_i(z) o_i(z,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial z^2}\left[\sigma_i^2(z) o_i(z,t)\right], \tag{19}$$

where $\bar{s}_i(t) = \int s_i(z,t) o_i(z,t)\, dz$ is the average saving rate in sector i, while sectoral wealth shares evolve according to

$$\dot{O}_i(t) = \left[\bar{s}_i(t) - \frac{\dot{A}(t)}{A(t)}\right] O_i(t). \tag{20}$$

This distinction is central because sectoral wealth shares determine specialization, and their evolution depends on sector-specific saving rates shaped by relative prices and the common interest rate.

3.2 Aggregated Productivity Dynamics

The model can also be used to decompose aggregate productivity dynamics into within-sector selection and between-sector reallocation components. In the closed economy, this decomposition shows that aggregate productivity gains are driven primarily by within-sector selection, while the between-sector component is quantitatively small and short-lived because relative prices adjust rapidly. Appendix E presents the formal decomposition and the results associated with the quantitative exercise in subsection 3.4.

3.3 Steady State

A steady state is an equilibrium in which sectoral wealth levels and wealth distributions are time-invariant. Formally, for all sectors $i \in I$,

$$\dot{A}_i(t) = 0, \quad g_i(a,z;t) = g_i(a,z), \quad \omega_i(z,t) = \omega_i(z), \quad \text{and} \quad O_i(t) = O_i. \tag{21}$$

In this environment, sectoral productivity and expenditure shares are constant, so specialization is characterized by the stationary allocation of wealth across sectors and productivity states. Appendix F complements the formal steady-state characterization with conditions for sectoral output, factor prices, capital income, and TFP.

3.4 Quantitative Illustration

This section studies how the introduction of sectoral structure alters the dynamics of wealth accumulation, selection, and aggregate productivity in a closed economy. To isolate the role of sectoral reallocation, I construct a one-sector benchmark and compare it to a two-sector economy initialized from the same underlying distributions of wealth and productivity.

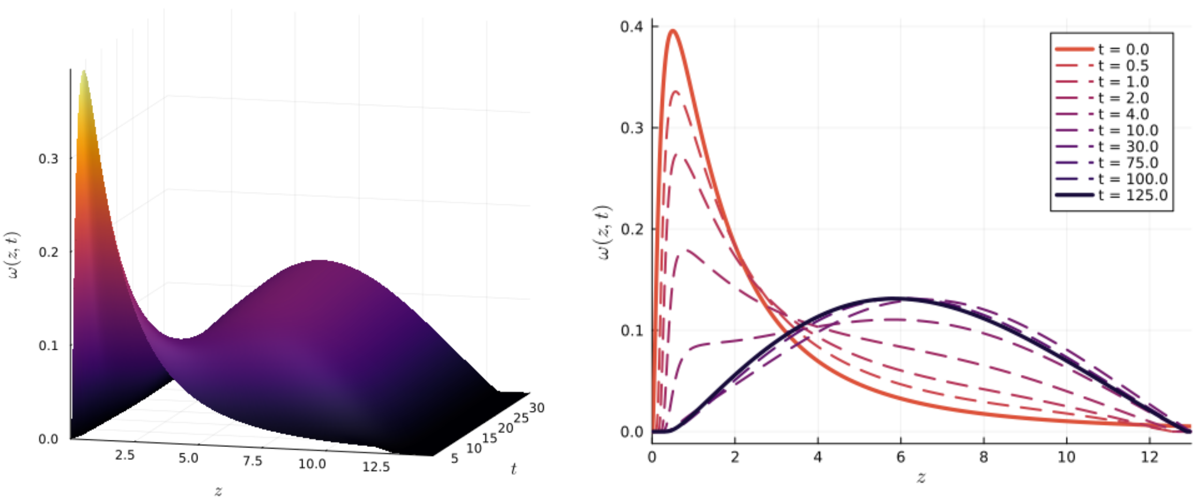
In the one-sector benchmark, I aggregate sectoral distributions into a single economy, so that aggregate initial wealth and the wealth-weighted productivity distribution coincide with those in the multisector environment. In the two-sector economy, I keep the same marginal distributions and total initial wealth, but allow for differences in the allocation of wealth across productivity levels across sectors.

This comparison isolates the role of sectoral structure as an additional margin of adjustment. While the one-sector economy features only within-sector selection and accumulation, the multisector economy allows for reallocation of wealth across sectors and endogenous changes in relative prices, which jointly shape the path of aggregate outcomes.

For the following exercises, I use a baseline parameterization summarized in Appendix I. Unless otherwise noted, parameters are common across sectors and chosen to generate a symmetric benchmark economy. Appendix H describes the numerical procedure used, including the fixed-point solution for equilibrium prices and the finite-difference approximation used to update the wealth-share distributions.

3.4.1 Benchmark: One-Sector Economy

I begin with a one-sector version of the model, which serves as a benchmark for the multisector analysis. In this environment, all entrepreneurs operate in the same sector and face the same production technology. Aggregate dynamics therefore reflect only within-sector heterogeneity, financial frictions, and the evolution of the wealth-weighted productivity distribution. There is no sectoral margin, so the model abstracts from changes in specialization.



(a) Initial periods

(b) Selected dates

Figure 2: One-sector wealth distribution dynamics.

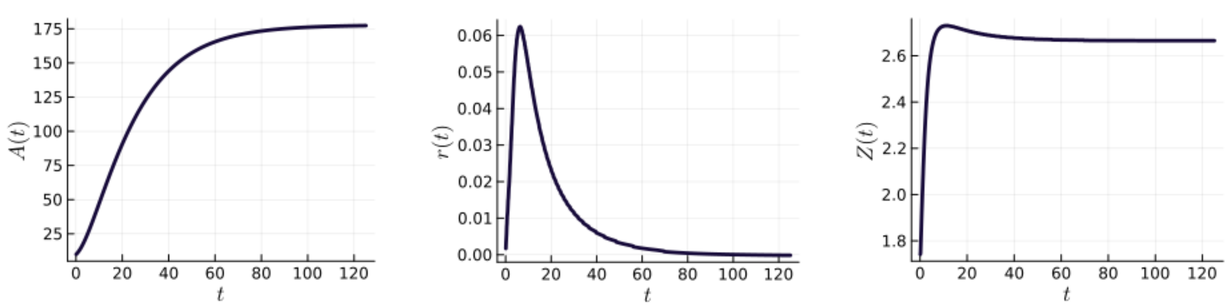
The figure shows the transition of the wealth-weighted productivity distribution. The one-sector economy isolates the standard within-sector selection mechanism.

Figure 2 illustrates the central selection mechanism inherited from Moll (2014). Entrepreneurs with higher productivity earn higher returns, save more, and gradually account for a larger share of aggregate wealth. Panel (a) shows this movement over the early transition: the wealth-weighted distribution shifts away from low productivity states and builds mass at higher productivity levels. Panel (b) shows the same process through selected cross-sections. The mass initially concentrated at low productivity declines, while the right side of the distribution becomes more important over time.

This within-sector reallocation affects aggregate outcomes. As wealth becomes increasingly concentrated among more productive entrepreneurs, aggregate productivity rises. At the same time, aggregate wealth accumulation reduces capital scarcity, which lowers the equilibrium interest rate and weakens the initial force of selection.

Figure 3 summarizes the corresponding aggregate dynamics. Aggregate wealth rises throughout the transition as entrepreneurs save and accumulate assets. The interest rate is high early in the transition, reflecting the high return to scarce capital, but declines as aggregate wealth increases. Aggregate productivity rises sharply at first because the economy reallocates wealth toward more productive entrepreneurs; over time, this process slows as the wealth distribution approaches its stationary shape.

The one-sector benchmark therefore captures the standard self-financing and selection mechanism: productive entrepreneurs gradually accumulate wealth, capital scarcity falls, and aggregate productivity improves. However, because there is only one sector, the benchmark has no notion of sectoral composition or specialization. The multisector economy adds this missing margin by allowing the same wealth-accumulation forces to operate both within sectors and across sectors through relative prices.



(a) Aggregate wealth (b) Interest rate (c) Aggregate productivity

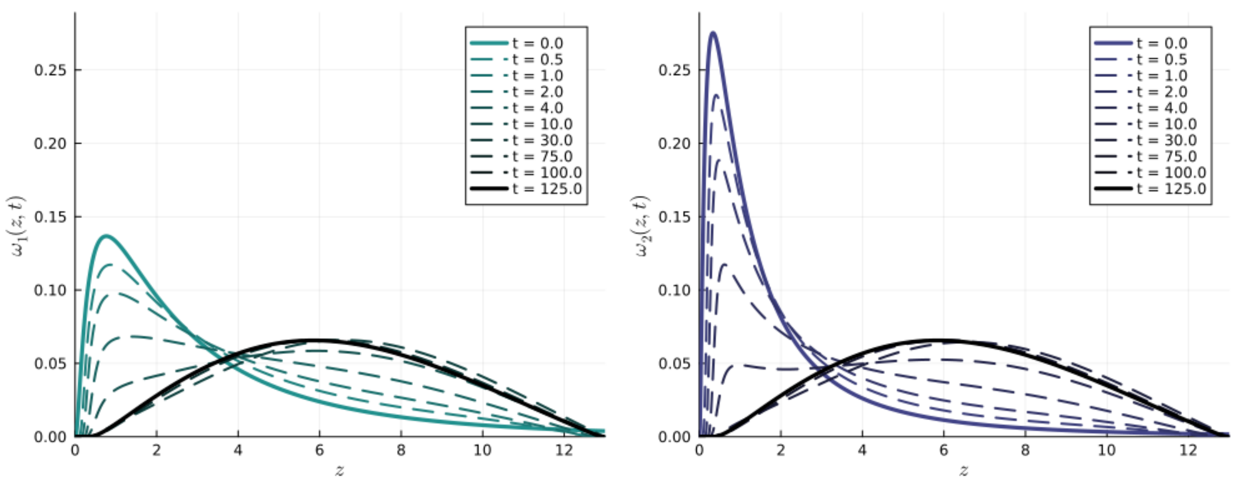
Figure 3: One-sector aggregate dynamics.

The figure shows the transition of aggregate wealth, the interest rate, and aggregate productivity. Early in the transition, the high return to capital reflects strong selection and reallocation toward productive entrepreneurs. As wealth accumulates, capital scarcity declines, the interest rate falls, and the transition becomes increasingly driven by aggregate accumulation.

3.4.2 Two-Sector Economy (Within- and Across-Sector Dynamics)

I now extend the analysis to a multisector economy with $i \in \{1, 2\}$, allowing for differences in initial wealth shares and productivity distributions across sectors. In particular, the initial correlation of log-wealth and log-productivity in sector 1 is initialized at $\rho(a, z)_{1,0} = 0.8$ and in sector 2 it is initialized at $\rho(a, z)_{1,0} = 0.2$. This introduces a new margin of adjustment, as resources can be reallocated both within and across sectors, and relative prices adjust endogenously to clear goods markets.

The comparison with the one-sector benchmark illustrates how sectoral structure modifies the dynamics of wealth accumulation, selection, and aggregate productivity.



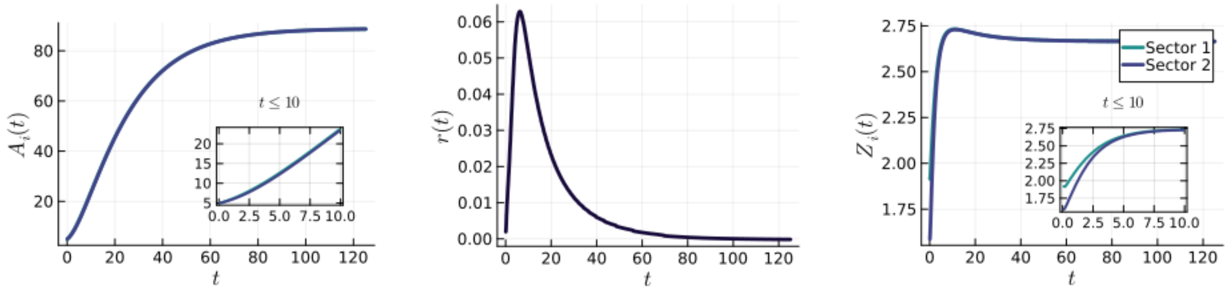
(a) Sector 1 (b) Sector 2

Figure 4: Total wealth sectoral shares distribution dynamics with two sectors.

The figure shows the transition of the sector-specific wealth-weighted productivity distributions in the two-sector closed economy. Differences in initial wealth-productivity alignment generate different short-run selection dynamics across sectors, but closed-economy price adjustment limits the persistence of these differences.

Within-sector selection. Figure 4 shows the evolution of the wealth-weighted productivity distributions $\omega_i(z, t)$ within each sector. As in the one-sector benchmark, more productive firms accumulate wealth and expand, leading to a gradual shift of mass toward higher productivity levels. However, the strength of this selection process differs across sectors, reflecting differences in initial conditions and equilibrium prices.

Sectoral aggregate dynamics The two-sector economy shows that the same selection forces present in the one-sector benchmark now operate alongside sectoral reallocation, which can be seen in Figure 5. At the aggregate level, the interest rate follows a familiar pattern: it rises sharply at the beginning, when productive entrepreneurs are expanding and capital is scarce, and then falls as wealth



(a) Aggregate wealth (b) Interest rate (c) Aggregate productivity

Figure 5: Two-sector aggregate dynamics.

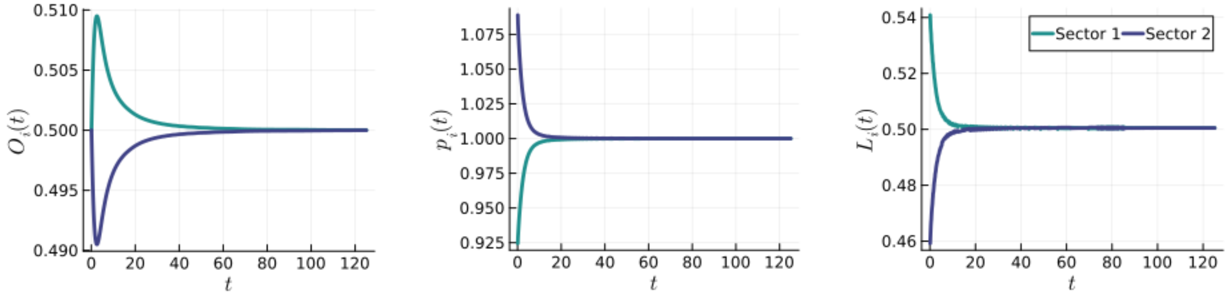
The figure shows transition dynamics in the two-sector closed economy. Panel (a) reports sectoral wealth levels $A_i(t)$, panel (b) reports the common interest rate $r(t)$, and panel (c) reports sectoral productivity $Z_i(t)$. The insets in panels (a) and (c) zooms in on the first ten periods to show the small early differences, especially between sectoral productivity paths, which are hard to see in the full-horizon plot. Colors identify sectors.

accumulates. Panel (c) shows that sectoral productivity increases rapidly in both sectors. The early gap between sectoral productivities is small and short-lived, reflecting the fact that closed-economy price adjustment compresses initial asymmetries. Overall, the figure illustrates that initial wealth-productivity differences matter for short-run sectoral dynamics, but general equilibrium forces limit their persistence in the closed economy.

Across-sector reallocation and prices. Figure 6 reports the evolution of sectoral wealth shares $O_i(t)$, output shares $Y_i(t)/Y(t)$, and relative prices $p_i(t)/p_j(t)$. Starting from asymmetric initial conditions, sectors with higher effective productivity attract a larger share of wealth and expand over time.

At the same time, this expansion induces general equilibrium effects through relative prices: as a sector grows, its relative price declines, reducing its profitability and diminishing further accumulation. This price adjustment mechanism partially offsets initial differences across sectors and slows down the reallocation of resources. Appendix E shows that the same force also limits the contribution of between-sector reallocation to aggregate productivity growth in the closed economy.

Taken together, these results show that sectoral structure introduces a reallocation margin that is absent in the one-sector benchmark, but that in a closed economy this margin is self-limiting: endogenous price adjustments act as a correction mechanism, reducing the persistence of initial asymmetries.



(a) Sectoral wealth shares (b) Sectoral prices (c) Labor composition

Figure 6: Across-sector reallocation.

The figure shows the transition dynamics of a two-sector closed economy with asymmetric initial wealth–productivity distributions. Panel (a) reports sectoral wealth shares $O_i(t)$, panel (b) sectoral prices $p_i(t)$, and panel (c) labor allocations $L_i(t)$. The sector with a more favorable initial allocation expands on impact, but general equilibrium price adjustments reduce differences in profitability, leading to convergence in allocations and selection thresholds over time.

4 Small Open Economy

The previous section characterized a closed economy in which relative prices are endogenously determined by domestic conditions. In that environment, sectoral dynamics are shaped jointly by wealth accumulation, financial frictions, and the endogenous adjustment of prices.

This section introduces trade through a small open economy with financial autarky. Goods prices are now taken as given from world markets, while factor prices and allocations remain determined domestically. This isolates the role of trade in shaping the interaction between wealth dynamics and sectoral allocation.

The analysis focuses on how exposure to international prices alters the evolution of sectoral productivity, wealth distributions, and aggregate outcomes. In particular, fixing relative prices eliminates a key adjustment margin present in the closed economy, modifying the feedback between sectoral profitability, savings, and capital allocation. Appendix J develops a simplified analytical benchmark illustrating how fixed world prices can sustain persistent specialization dynamics in a small open economy environment.

4.1 Setup and Equilibrium

The economy is populated by entrepreneurs facing the same production and financial frictions as in the closed economy. The key difference is that sectoral goods prices $\{p_i^*(t)\}_{i\in I}$ are exogenously given by world markets.

Definition 7 (Small open economy equilibrium with financial autarky)**.** Given initial wealth distributions $\{\omega_i(z,0)\}_{i\in I}$, wealth levels $\{A_i(0)\}_{i\in I}$, and a sequence of world prices $\{p_i^*(t)\}_{i\in I,t\geq 0}$, a competitive small open economy equilibrium with financial autarky is a path of factor prices

$$\{w(t), r(t)\}_{t\geq 0},$$

allocation rules

$$\{k_i(z,t), \ell_i(z,t), c_i(z,t)\}_{i\in I, t\geq 0},$$

net exports

$$\{NX_i(t)\}_{i\in I, t\geq 0},$$

and wealth distributions

$$\{\omega_i(z,t)\}_{i\in I, t\geq 0},$$

such that:

1. Entrepreneurs optimize given world prices and factor prices.

2. Factor markets clear:

$$\bar{L} = A(t)\sum_{i\in I}\int \ell_i(z,t)\,\omega_i(z,t)\,dz,$$
$$A(t) = A(t)\sum_{i\in I}\int k_i(z,t)\,\omega_i(z,t)\,dz.$$

3. Goods markets satisfy

$$Y_i(t) = D_i(t) + NX_i(t), \qquad \forall i \in I,$$

   where domestic absorption satisfies the CES demand system

$$D_i(t) = \upsilon_i \left(\frac{p_i^*(t)}{P^*(t)}\right)^{-\eta} D(t),$$

   with price index

$$P^*(t)^{1-\eta} = \sum_{i\in I} \upsilon_i \left(p_i^*(t)\right)^{1-\eta}.$$

4. External balance holds:

$$\sum_{i\in I} p_i^*(t) NX_i(t) = 0.$$

5. Aggregate absorption satisfies

$$P^*(t)D(t) = P^*(t)C(t) + \dot{A}(t) + \delta A(t),$$

where aggregate consumption expenditure is

$$P^*(t)C(t) = w(t)\bar{L} + \rho A(t).$$

6. Wealth distributions evolve consistently with individual wealth accumulation and the productivity process.

4.2 Persistence Mechanisms

The quantitative exercises in this section should be read as mechanism illustrations rather than a calibration. The baseline parametrization is the same used in subsection 3.4 and summarized in Appendix I. I keep technologies, preferences, and aggregate resources fixed, and vary either the initial wealth-productivity distribution or the world-price regime. This isolates the channel emphasized by the model: when relative prices are fixed, initial differences in effective sectoral capital are not offset by domestic price adjustment and can therefore persist through wealth accumulation.

The results in this section reinterpret standard trade-theory forces in a setting where effective factor supplies are endogenous to wealth distributions. In the standard Rybczynski theorem, changes in factor endowments shift production patterns at fixed goods prices. Here, sectoral wealth and the within-sector allocation of wealth across productivity types play an analogous role: they determine how much effective capital a sector can deploy under collateral constraints. In addition, these effective endowments evolve through saving and wealth accumulation.

Proposition 8. *Consider a small open economy with financial autarky and exogenous goods prices $\{p_i^*\}_{i \in I}$.*

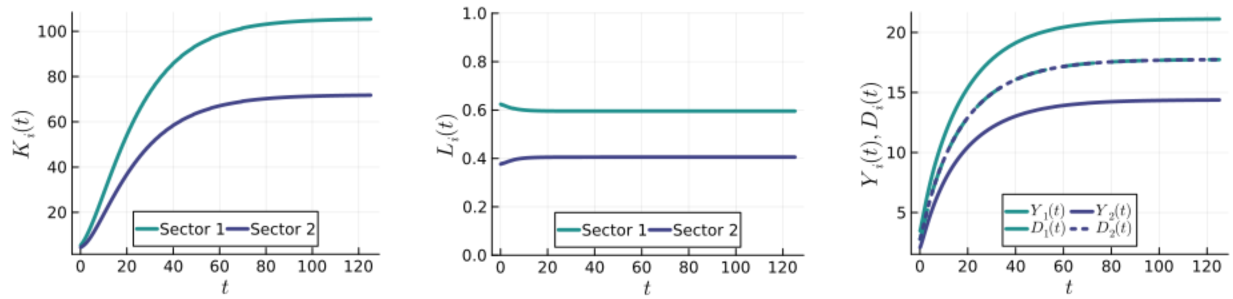
(i) Wealth effect. *Holding fixed the within-sector distribution $o_i(z,t)$, an increase in sectoral wealth $A_i(t)$ raises capital employed, labor demand, and output in sector i:*

$$\frac{\partial K_i(t)}{\partial A_i(t)} > 0, \qquad \frac{\partial L_i(t)}{\partial A_i(t)} > 0, \qquad \frac{\partial Y_i(t)}{\partial A_i(t)} > 0.$$

(ii) Allocation effect. *Holding fixed sectoral wealth $A_i(t)$, a shift in $o_i(z,t)$ that raises sectoral productivity $Z_i(t)$ increases effective capital use and output in sector i:*

$$\frac{\partial K_i(t)}{\partial Z_i(t)} > 0, \qquad \frac{\partial Y_i(t)}{\partial Z_i(t)} > 0.$$

At given world prices, sectoral wealth and within-sector allocation both operate as sources of effective factor endowments. Higher wealth allows entrepreneurs to scale up production under collateral constraints, while a more favorable allocation toward high-productivity entrepreneurs raises the efficiency with which capital is used. As a result, sectors with larger wealth stocks or better within-sector allocation expand relative to others, reallocating labor and capital across sectors. The model therefore delivers a distributional analogue of the Rybczynski theorem, in which changes in wealth and allocation play the role of factor endowments in shaping production patterns.



(a) Capital (b) Labor (c) Output and Absorption

Figure 7: Sectoral allocation dynamics at given world prices.

The figure shows the transition dynamics of sectoral allocations in a small open economy with fixed world prices. Panel (a) reports capital by sector $K_i(t)$, panel (b) labor allocations $L_i(t)$, and panel (c) output $Y_i(t)$ together with sectoral absorption $D_i(t)$ (dashed lines). Because goods prices are exogenous, differences in initial wealth–productivity distributions translate into persistent differences in capital, labor, and output across sectors. In contrast to the closed economy, the absence of relative price adjustment prevents reallocation forces from equalizing sectoral outcomes over time.

Proposition 9 (Dynamic Rybczynski effect through wealth accumulation)**.** *Consider a small open economy with financial autarky and exogenous goods prices $\{p_i^*\}_{i\in I}$, and let sectoral wealth shares be $O_i(t) = A_i(t)/A(t)$.*

Suppose that sector i has either:

1. *a higher initial wealth level $A_i(0)$, holding fixed $o_i(z,0)$; or*
2. *a more favorable initial within-sector allocation $o_i(z,0)$ that raises measured productivity $Z_i(0)$.*

Then, at given world prices, sector i is larger on impact:

$$K_i(0)\uparrow, \qquad L_i(0)\uparrow, \qquad Y_i(0)\uparrow.$$

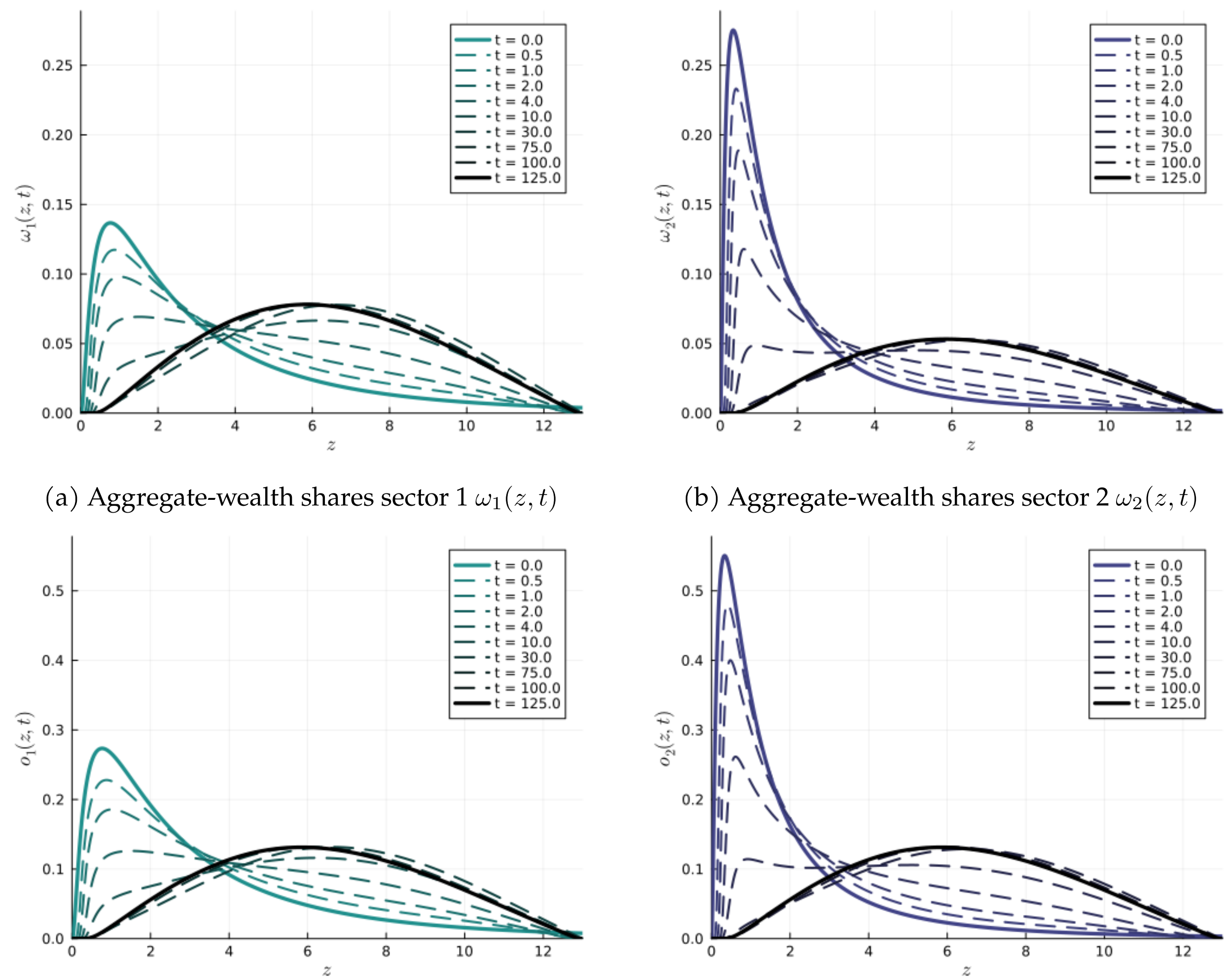
If these initial conditions imply a higher average net saving rate in sector i relative to the aggregate, then

$$\frac{d}{dt}\log O_i(t) > 0$$

locally in time.

Consequently, sector i accumulates wealth faster along the transition dynamics, increasing its relative size over time. In a small open economy, fixed world prices prevent the relative-price adjustment that would otherwise reduce these initial asymmetries, allowing them to persist in the long run.

Initial advantages in wealth or within-sector allocation translate into higher profitability and higher saving rates when goods prices are fixed. Because entrepreneurs face collateral constraints, higher wealth allows productive agents to expand, reinforcing the initial advantage through accumulation. As a result, sectors that are initially larger or better allocated grow faster and continue to attract resources over time. This generates a dynamic amplification mechanism: differences in initial conditions are not corrected, but instead propagate through wealth accumulation, shaping specialization patterns over the entire transition. Appendix J presents a stripped-down analytical benchmark showing how fixed world prices can generate persistent specialization through sector-specific capital accumulation.



(a) Aggregate-wealth shares sector 1 $\omega_1(z,t)$

(b) Aggregate-wealth shares sector 2 $\omega_2(z,t)$

(c) Within-sector-wealth shares sector 1 $o_1(z,t)$

(d) Within-sector-wealth shares sector 2 $o_2(z,t)$

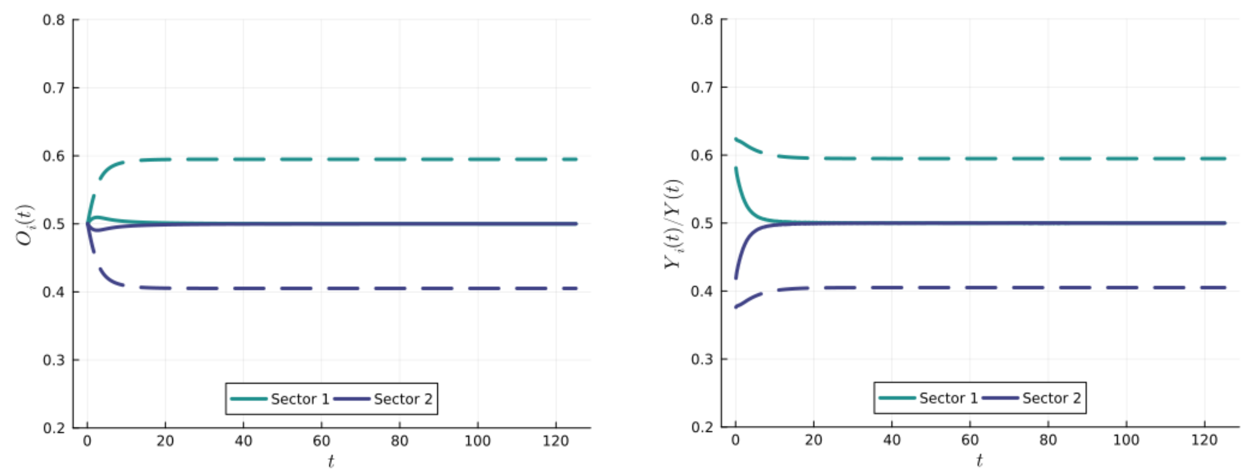
Figure 8: Wealth distribution dynamics at fixed world prices.

The figure shows the evolution of wealth distributions in a two-sector small open economy with fixed world prices. Panels (a) and (b) report the aggregate wealth distributions $\omega_i(z,t)$, which combine sectoral size and within-sector allocation, while panels (c) and (d) report the within-sector distributions $o_i(z,t)$, isolating selection effects.

Wealth distribution dynamics As represented in Figure 8, within each sector, the distribution shifts toward higher productivity types over time, reflecting selection driven by differential returns and savings behavior. However, because goods prices are exogenous, these within-sector forces do not translate into full reallocation across sectors. As a result, differences in initial wealth–productivity distributions generate persistent gaps in sectoral size, visible in the divergence of $\omega_1(z,t)$ and $\omega_2(z,t)$, even as the within-sector distributions $o_i(z,t)$ converge to a common stationary distribution. This illustrates how, in a small open economy, the absence of relative price adjustment limits the equalizing effects of reallocation and sustains long-run differences across sectors.

4.3 Comparison with Closed Economy

To isolate the role of price adjustment, I compare the transition dynamics in the closed economy and the small open economy starting from identical initial wealth–productivity distributions. This comparison holds fixed fundamentals and initial conditions, so differences in outcomes reflect only the behavior of prices across regimes. In the closed economy, relative prices respond to sectoral expansion and reallocation, reducing initial asymmetries. In the small open economy, prices are fixed, removing this adjustment margin. As a result, the two environments generate markedly different dynamics, with convergence in the closed economy and persistent differences in sectoral allocation in the open economy.



(a) Wealth shares: closed vs SOE

(b) Output shares: closed vs SOE

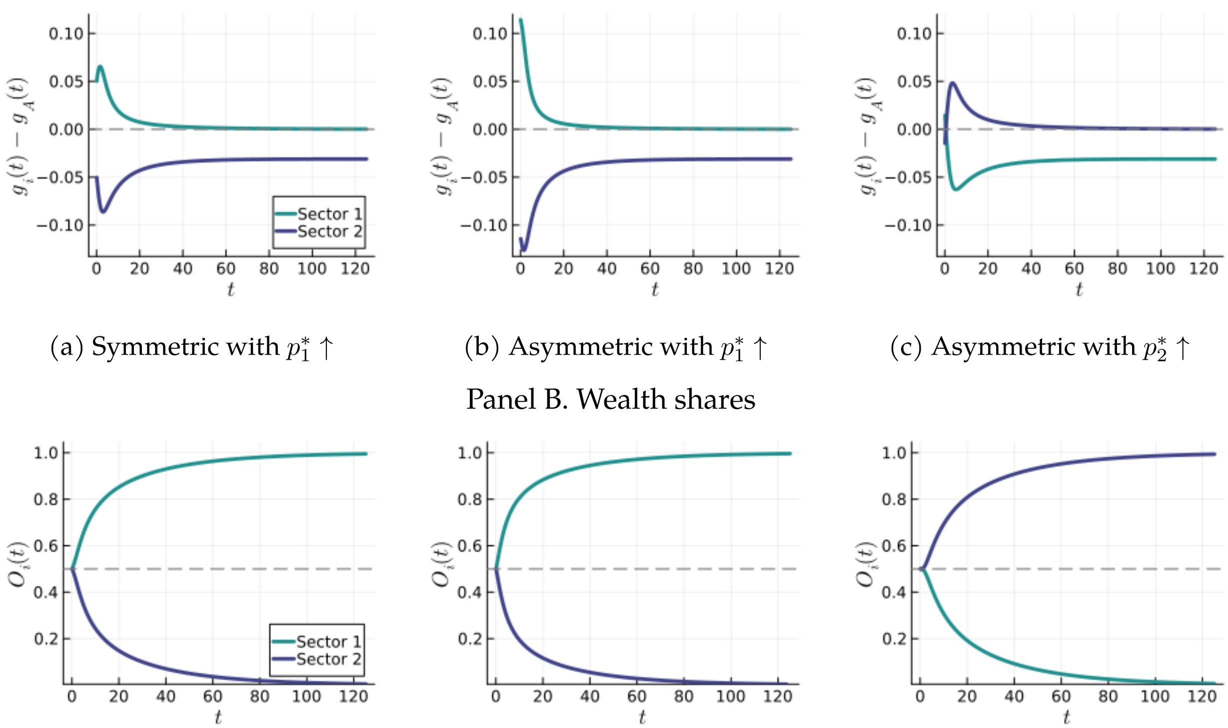
Figure 9: Adjustment and persistence across regimes.

The figure compares sectoral allocation dynamics in a closed economy and a small open economy initialized from the same wealth–productivity distributions. Panel (a) reports sectoral wealth shares $O_i(t)$, and panel (b) reports output shares $Y_i(t)/Y(t)$. Colors identify sectors. Solid lines correspond to the closed economy, while dashed lines correspond to the small open economy.

4.4 Price shocks and endogenous specialization

Having shown that fixed prices generate persistence by removing the adjustment margin present in the closed economy, I now study how exogenous price changes affect sectoral dynamics in the open economy. In this environment, world prices act as external drivers of profitability, shifting incentives for accumulation across sectors. Because capital allocation is mediated by wealth and financial frictions, these price changes translate into differential saving rates and, in turn, into changes in sectoral wealth shares over time. The response therefore depends not only on the direction of the price shock, but also on the underlying distribution of wealth within sectors. Corollary 10 formalizes this mechanism, and Figure 10 illustrates how price shocks generate dynamic reallocation and persistent changes in sectoral composition.

Panel A. Saving-rate gap



(a) Symmetric with $p_1^* \uparrow$ (b) Asymmetric with $p_1^* \uparrow$ (c) Asymmetric with $p_2^* \uparrow$

Panel B. Wealth shares

(d) Symmetric with $p_1^* \uparrow$ (e) Asymmetric with $p_1^* \uparrow$ (f) Asymmetric with $p_2^* \uparrow$

Figure 10: Dynamic response to world price shocks.

The figure reports the response of a small open economy to permanent changes in world prices. Panel A plots sectoral saving-rate gaps $g_i(t) - g_A(t)$, and Panel B reports sectoral wealth shares $O_i(t)$. Colors identify sectors. Columns correspond to symmetric and asymmetric environments, with price increases in sector 1 (columns (a)–(b)) and sector 2 (column (c)). In each case, the sector facing the higher price exhibits a positive saving-rate gap, while the other exhibits a negative gap, with corresponding changes in wealth shares.

Corollary 10 (Dynamic price accumulation channel)**.** *Consider a small open economy with financial autarky and exogenous goods prices $\{p_i^*\}_{i\in I}$. Let sectoral wealth shares evolve according to*

$$\frac{d}{dt}\log O_i(t) = g_i(t) - g_A(t),$$

where $g_i(t) = \int s_i(z,t)\, o_i(z,t)\, dz$ is the average net saving rate in sector i, and $g_A(t)$ is the aggregate counterpart.

A permanent increase in the world price of good i, $p_i^* \uparrow$, that raises sectoral profitability and the associated saving rate implies

$$\frac{\partial}{\partial p_i^*}\left(\frac{d}{dt}\log O_i(t)\right) > 0.$$

Moreover, the magnitude of this response depends on the within-sector distribution $o_i(z,t)$: for a given price change, a higher concentration of wealth among high-productivity entrepreneurs increases $g_i(t)$ and strengthens the response of $O_i(t)$.

Interpretation. Changes in goods prices affect sectoral dynamics through their impact on profitability and saving behavior. A higher price for a given sector increases the returns to capital in that sector, raising its average saving rate relative to the aggregate. This generates a positive growth differential in wealth shares, leading the sector to expand over time. Because saving rates depend on the distribution of wealth across entrepreneurs, the response is amplified when wealth is concentrated among more productive agents. As a result, price changes do not only affect contemporaneous allocation, but also the evolution of wealth and the path of specialization through time.

4.5 What does trade do in this model?

Trade changes the role of prices in the adjustment process. In the closed economy, sectoral expansion feeds back into relative prices, which move against the expanding sector and reduce its profitability. This price adjustment limits the persistence of initial differences in wealth and productivity by diminishing the incentives for further accumulation. In contrast, in the small open economy, goods prices are fixed at world levels. As a result, sectoral profitability does not adjust in response to domestic reallocation, and initial differences in wealth and productivity translate directly into differences in growth rates and sectoral expansion.

Because capital allocation is mediated by balance sheets, trade does not eliminate distortions. Instead, it reshapes them. Sectors with initially higher wealth or more favorable within-sector allocation experience higher returns and accumulate wealth faster, leading to persistent differences in specialization. Financial frictions therefore interact with trade by amplifying the effects of initial conditions. While trade integration reallocates production toward sectors with higher world prices, the extent of this reallocation depends on the distribution of wealth, so economies with similar fundamentals but different initial distributions can follow different specialization paths over time.

5 Empirical Evidence

The divergence in sectoral specialization documented in the introduction points to the importance of dynamic forces shaping comparative advantage. This section provides firm-level evidence on the mechanism, focusing on two facts: the presence of persistent productivity dispersion within sectors, and the relationship between productivity and firm growth.

5.1 Data

The empirical analysis combines firm-level survey data from Peru with international trade data from BACI International Trade Database (Gaulier & Zignago, 2010). The analysis uses firm-level data from the Annual Economic Survey (*Encuesta Económica Anual*, EEA), conducted by Instituto Nacional de Estadística e Informática (INEI) (2001-2019). The survey is administered annually for national accounts and sectoral analysis and is mandatory for all firms above a minimum sales threshold, effectively covering the universe of medium and large formal firms. The data include detailed balance-sheet and income-statement information, such as sales, value added, costs, exports, employment, wages, and capital stocks. Coverage spans all major sectors except mining and extractive industries, which report to a different administrative source.

International trade data are constructed using bilateral trade flows from BACI at the HS6-product level for the period 1995–2024. Products are mapped to ISIC Revision 3 sectors using concordance tables, and sector-level measures of external demand, supply, trade exposure, and unit-value price proxies are constructed using trade flows excluding Peru.

The combined dataset allows sectoral trade shocks to be linked to firm-level dynamics in productivity, capital accumulation, and sectoral specialization patterns over time.

Figure 11 shows that firm-level productivity exhibits substantial dispersion within sectors. The distributions are wide and persistent over time, and their dispersion varies systematically across sectors. These patterns indicate that sectors are characterized by stable heterogeneity in firm-level productivity, providing scope for selection and reallocation dynamics.

Table 1 shows that more productive firms expand faster, but that this relationship varies substantially across sectors. In the baseline specification, lagged productivity is positively associated with both capital and labor growth, consistent with selection driven by higher returns and wealth accumulation.

However, the strength of this relationship differs across sectors. Relative to agroindustry, most sectors exhibit a weaker response of capital growth to productivity, while a few sectors display a stronger relationship. For labor growth, the heterogeneity is even more pronounced: some sectors show attenuated or even negative responses, while others, such as manufacturing and fishing, exhibit a strong positive relationship between productivity and expansion.

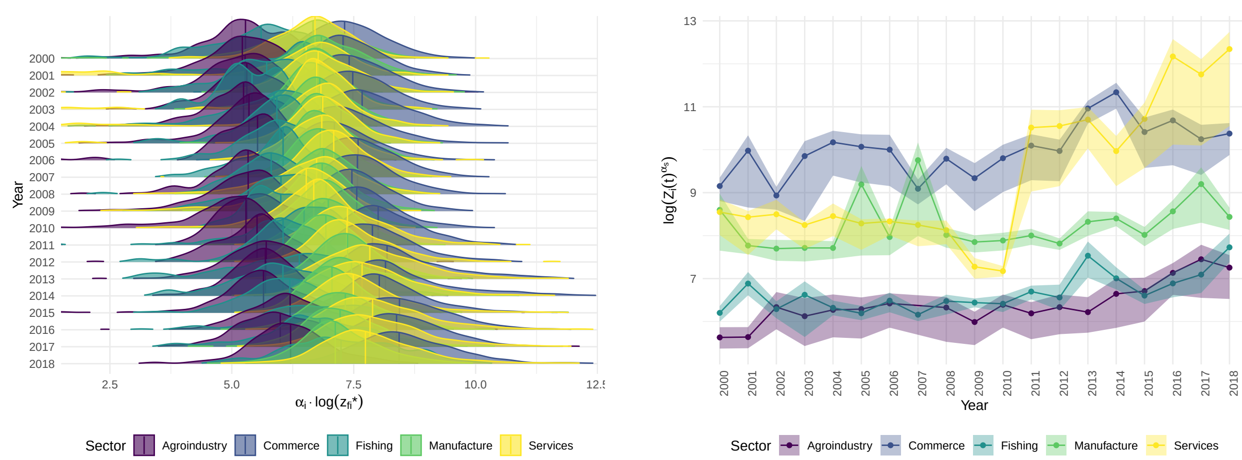
These patterns are consistent with a setting in which sectoral environments shape the extent to which productive firms can scale up. Differences in the responsiveness of growth to productivity

Table 1: Productivity driven expansion differs across sectors.

	Capital growth		Labor growth	
	Coef.	SE	Coef.	SE
Baseline (Ref.: Agroindustry)				
Lagged productivity	0.081***	(0.004)	0.194***	(0.011)
Sector interactions with lagged productivity				
Commerce	-0.042***	(0.001)	-0.068***	(0.005)
Construction	-0.034***	(0.004)	0.026**	(0.007)
Electricity and services	-0.115***	(0.004)	-0.153***	(0.005)
Hydrocarbons and mining	-0.047***	(0.003)	-0.185***	(0.004)
Manufacturing	-0.030***	(0.002)	0.187***	(0.002)
Fishing	0.083***	(0.004)	0.099***	(0.007)
Services	-0.039***	(0.003)	-0.044***	(0.002)
Observations	22,942		22,942	
R-squared	0.032		0.132	

Firm-level regressions of capital and labor growth on lagged productivity and its interaction with sector fixed effects. All specifications include sector and year fixed effects. Standard errors clustered at the sector level in parentheses. *** p<0.01, ** p<0.05, * p<0.10.

suggest that the process of reallocation toward high-productivity firms is uneven across sectors. In the context of the model, this heterogeneity can be interpreted as reflecting differences in the interaction between wealth distributions and financial frictions, which affect the ability of productive



(a): Empirical distr. of z^{α}

(b): TFPR weighted by capital

Figure 11: Distribution of firm-level productivity by sector.

For both panels, productivity is constructed as a residual $\alpha_i \log z^* fi = \log(p_i y fi) - \alpha_i \log k_{fi} - (1-\alpha_i) \log l_{fi}$. a) Calculated using product-capital elasticities variables by sector: $\alpha_{\text{Agroindustry}} = 0.452$, $\alpha_{\text{Fishing}} = 0.417$, $\alpha_{\text{Manufacturing}} = 0.343$, $\alpha_{\text{Services}} = 0.328$, estimated with IV. b) Trends show capital-weighted sample estimates of $\log(Z_i) = \log\left(\mathbb{E}[z_i(t)]^{\alpha_i}\right)$ (log-TFPR). Bands indicate 95% bootstrapped confidence intervals.

firms to expand and accumulate capital over time.

5.2 Empirical Strategy

The model emphasizes a dynamic amplification mechanism through which sectoral shocks affect wealth accumulation, which in turn shapes future specialization patterns. In particular, sectors exposed to favorable price or demand shocks experience higher profitability and saving rates, allowing productive firms to accumulate wealth and expand persistently over time.

This subsection outlines an empirical strategy to discipline this mechanism using sector-level trade shocks and firm-level balance sheet data.

5.2.1 Trade shocks and sectoral exposure indicators

The empirical strategy uses international trade data excluding Peru to construct measures of changes in external conditions across sectors. The main object of interest is a measure of external demand conditions that allows sectoral exposure to evolve over time, consistent with the endogenous specialization dynamics emphasized in the model. I also construct complementary measures based on fixed exposure shares, international unit values, and world export growth.

The baseline measure used throughout subsection 5.3 captures changes in external demand faced by Peruvian sectors while allowing exposure shares to vary over time:

$$DynamicDemandShock_{i,t} = \sum_{d} \omega_{id,t-1} \left(\log M_{d,i,t}^{World \setminus Peru} - \log M_{d,i,t-1}^{World \setminus Peru} \right),$$

where $\omega_{id,t-1}$ denotes lagged export exposure of sector i to destination market d, and $M_{d,i,t}^{World \setminus Peru}$ denotes imports by destination d in sector i from the rest of the world excluding Peru.

This specification captures changes in external demand conditions weighted by the evolving export composition of Peruvian sectors. In the model, sectoral exposure changes endogenously through wealth accumulation and specialization dynamics: sectors experiencing favorable profitability conditions gradually expand and become more exposed to growing destination markets. The dynamic exposure measure therefore incorporates both changes in foreign demand and endogenous amplification through evolving specialization patterns.

For comparison, I also construct a conventional shift-share measure using fixed initial exposure shares:

$$DemandShock_{i,t} = \sum_{d} \omega_{id,0} \left(\log M_{d,i,t}^{World \setminus Peru} - \log M_{d,i,t-1}^{World \setminus Peru} \right),$$

where $\omega_{id,0}$ denotes initial destination exposure shares. This specification isolates variation arising from changes in foreign import demand while abstracting from contemporaneous changes in Peru's export composition.[4]

To proxy for changes in international prices, I additionally construct sector-level unit-value shocks. For each HS6 product h, define the foreign unit value as

$$uv_{h,t}^{World\setminus Peru} = \frac{\sum_{o\neq Peru}\sum_d v_{o,d,h,t}}{\sum_{o\neq Peru}\sum_d q_{o,d,h,t}},$$

where $v_{o,d,h,t}$ and $q_{o,d,h,t}$ denote the value and quantity of exports from origin o to destination d in product h and year t. Sector-level price changes are then constructed as

$$PriceShock_{i,t} = \sum_{h\in i} \omega_{ih,0} \left(\log uv_{h,t}^{World\setminus Peru} - \log uv_{h,t-1}^{World\setminus Peru} \right).$$

This measure captures changes in international unit values faced by Peruvian sectors. Since unit values may also reflect quality upgrading, compositional changes, and measurement error, I interpret this object as a proxy for external price conditions rather than a direct measure of prices.

Finally, I construct a measure of external supply conditions based on export growth in the rest of the world:

$$SupplyShock_{i,t} = \Delta \log X_{i,t}^{World\setminus Peru},$$

where $X_{i,t}^{World\setminus Peru}$ denotes exports by the rest of the world in sector i.

Together, these measures provide complementary indicators of changes in external conditions. Dynamic demand shocks capture evolving exposure to foreign demand growth, fixed-weight demand shocks isolate exogenous variation in destination demand, price shocks proxy for changes in international profitability conditions, and supply shocks capture changes in competitive pressure from the rest of the world.

5.2.2 Empirical Specifications and Mechanism Discipline

This subsection outlines a set of empirical specifications designed to connect the model's dynamic mechanism to observed firm- and sector-level responses in the data. The goal is not to estimate static treatment effects, but rather to evaluate whether external trade shocks generate persistent changes in accumulation, reallocation, and specialization patterns consistent with the mechanism emphasized in the model.

The specifications focus on three related objects: firm-level expansion following sectoral trade shocks, the evolution of sectoral shares over time, and the persistence of specialization patterns through endogenous wealth accumulation. A central prediction of the model is that sectors exposed to favorable external conditions should experience stronger and more persistent expansion

[4] A useful special case is the expansion in Chinese import demand following China's accession to the WTO, which can be interpreted as a large realization of the broader external-demand framework considered here.

when productive firms are able to accumulate wealth and scale up under financial frictions.

Firm expansion and external demand shocks. The model predicts that favorable external demand conditions raise sectoral profitability and affect firm expansion heterogeneously within sectors. Because firms face collateral constraints, more productive firms are better positioned to translate improved external demand conditions into subsequent capital and labor expansion.

To evaluate this mechanism, I combine firm-level panel data from the Peruvian *Encuesta Económica Anual* (EEA) with sector-level measures of external demand shocks constructed from international trade data. The empirical specification relates one-period firm growth to lagged sectoral shocks and lagged firm productivity:

$$\Delta \log x_{f,i,t} = \beta_\ell Shock_{i,t-\ell} + \gamma_\ell z_{f,i,t-1} + \delta_\ell Shock_{i,t-\ell} \times z_{f,i,t-1} + \mu_i + \lambda_t + \varepsilon_{f,i,t},$$

where $\Delta \log x_{f,i,t}$ denotes one-period firm growth in capital or employment, $Shock_{i,t-\ell}$ is the sector-level external demand shock lagged by ℓ years, and $z_{f,i,t-1}$ is lagged firm productivity. I estimate the specification separately for lags $\ell = 1, \dots, 5$. All specifications include sector and year fixed effects, and standard errors are clustered at the sector level.

The interaction term captures a central implication of the model: favorable external demand conditions should disproportionately affect productive firms, which are better able to expand under financial constraints. The estimates therefore test whether productive firms in sectors exposed to stronger past external demand shocks experience faster subsequent growth in capital and labor.

Persistence and sectoral specialization. A central prediction of the model is that trade shocks affect not only contemporaneous allocation, but also future specialization patterns through endogenous accumulation dynamics. To evaluate the persistence of sectoral responses over time, I estimate local-projection specifications relating future changes in sectoral shares to current external trade shocks.

Specifically, for each horizon $h \geq 0$, consider regressions of the form

$$\Delta_h \log O_{i,t+h} = \beta_h Shock_{i,t} + \phi_h \Delta \log A_{i,t} + \mu_i + \lambda_t + \varepsilon_{i,t+h},$$

where

$$\Delta_h \log O_{i,t+h} = \log O_{i,t+h} - \log O_{i,t},$$

$O_{i,t}$ denotes sectoral output, export, or employment shares, and $A_{i,t}$ measures accumulated sectoral assets or capital.

The sequence of coefficients $\{\beta_h\}_{h\geq 0}$ traces the dynamic response of sectoral specialization to external shocks over time. In the model, sectors receiving favorable trade shocks experience persistent increases in sectoral shares because higher profitability induces wealth accumulation and gradual expansion among productive firms.

More generally, the framework implies that temporary external shocks may generate long-lasting differences in specialization patterns through endogenous accumulation and reallocation dynamics. Comparing the persistence of estimated responses in the data and in simulated model economies therefore provides a direct way to discipline the mechanism emphasized in the model.

Discussion. Taken together, these specifications are designed to discipline the model's dynamic mechanism using both firm-level and sector-level evidence. The key object of interest is the persistence of sectoral responses over time and their relationship with wealth accumulation, productivity, and reallocation.

More broadly, the empirical analysis seeks to evaluate whether external trade shocks generate persistent differences in specialization through balance-sheet dynamics, as predicted by the model.

5.3 Results

Table 2 reports firm-level regressions relating subsequent firm growth to lagged external demand shocks, lagged productivity, and their interaction. Across specifications, lagged productivity is strongly associated with subsequent capital and labor growth, consistent with persistent heterogeneity in firm expansion dynamics. More importantly, the interaction between external demand shocks and lagged productivity is positive in several specifications, indicating that productive firms tend to expand more strongly following favorable external demand conditions. This pattern is consistent with the mechanism emphasized in the model, in which productive firms are better able to translate improved profitability conditions into subsequent expansion under financial frictions.

The estimates also suggest that the effects of external demand conditions are heterogeneous across firms and evolve gradually over time. In several specifications, the direct effect of the shock is small or negative, while the interaction with productivity remains positive. This pattern is consistent with reallocation dynamics within sectors, where favorable external conditions disproportionately benefit productive firms rather than generating uniform expansion across all producers.

Figure 12 reports local-projection estimates of the dynamic response of sectoral employment shares to external demand shocks. The estimates display relatively flat pre-trends and positive post-shock responses over subsequent years, suggesting that favorable external demand conditions are associated with persistent changes in sectoral employment composition. Although the responses are gradual and somewhat noisy across horizons, the overall pattern is consistent with the idea that temporary external shocks can generate persistent specialization dynamics through endogenous accumulation and reallocation mechanisms.

Taken together, these results provide preliminary empirical support for the paper's central mechanism. External demand shocks appear to affect both firm-level expansion and medium-run sectoral reallocation in ways consistent with wealth-dependent accumulation dynamics under financial frictions. Alternative results that use a demand shock measure using fixed initial exposure

Table 2: External demand shocks and firm expansion

	Panel A. Capital growth				
	ℓ=1	ℓ=2	ℓ=3	ℓ=4	ℓ=5
Dynamic demand shock	-0.202	-0.153	-0.237	-0.153***	-0.066
	(0.146)	(0.121)	(0.233)	(0.044)	(0.062)
Lagged productivity	0.065***	0.064***	0.063***	0.062***	0.062***
	(0.010)	(0.010)	(0.011)	(0.011)	(0.011)
Shock × productivity	0.027	0.027*	0.018	0.031**	-0.001
	(0.020)	(0.015)	(0.033)	(0.013)	(0.020)
Observations	4,521	4,521	4,521	4,521	4,521
	Panel B. Labor growth				
	ℓ=1	ℓ=2	ℓ=3	ℓ=4	ℓ=5
Dynamic demand shock	-0.250**	-0.004	-0.278	-0.163	0.108
	(0.094)	(0.189)	(0.414)	(0.131)	(0.084)
Lagged productivity	0.067***	0.065***	0.064***	0.064***	0.065***
	(0.018)	(0.019)	(0.017)	(0.017)	(0.018)
Shock × productivity	0.033**	0.020	0.063	0.040	-0.026
	(0.013)	(0.025)	(0.056)	(0.025)	(0.024)
Observations	4,532	4,532	4,532	4,532	4,532

Notes: The table reports firm-level regressions of capital and labor growth on lagged sector-level dynamic demand shocks, lagged firm productivity, and their interaction. Panel A uses capital growth as the dependent variable, and Panel B uses labor growth. Columns correspond to shock lags from one to five years. Dynamic demand shocks are constructed from changes in external import demand using lagged Peruvian export shares as exposure weights. Lagged productivity is measured using the firm-level productivity residual described in the text. All regressions include sector and year fixed effects. Standard errors are clustered at the sector level. Significance levels are denoted by * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$.

shares are available in Appendix K.

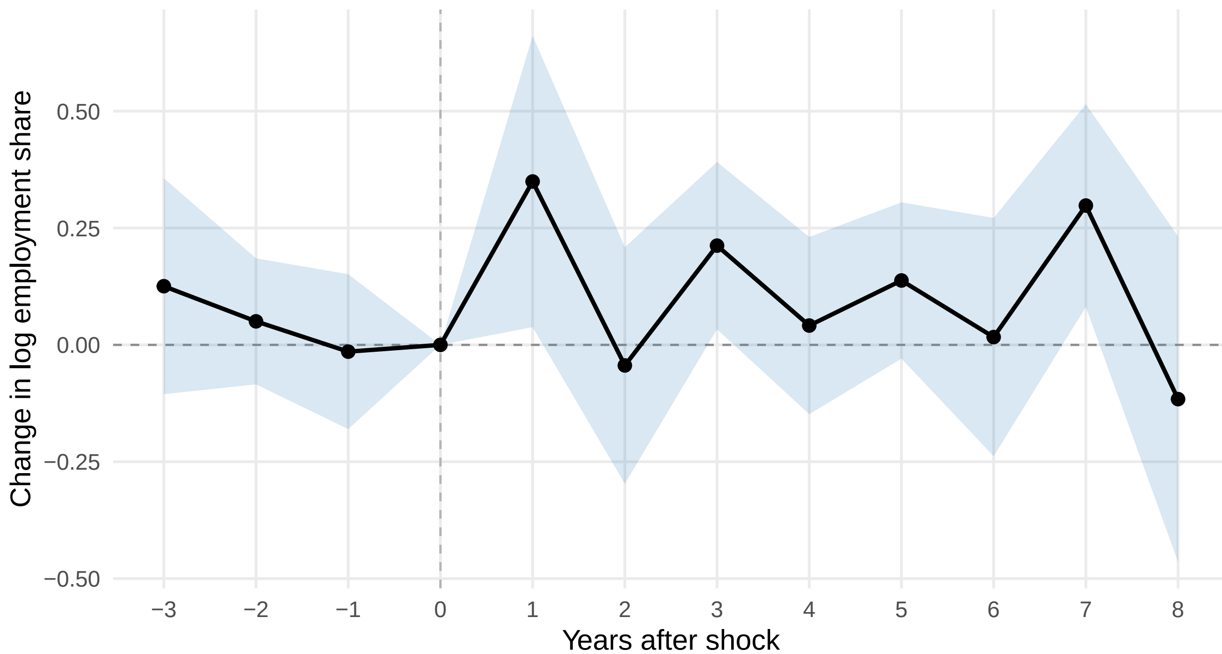


Figure 12: Dynamic Response of Sectoral Employment Shares to External Demand Shocks

Notes: The figure plots local-projection estimates of the dynamic response of sectoral employment shares to external demand shocks in Peru. The horizontal axis reports years relative to the shock, where negative values correspond to pre-trends and positive values to post-shock responses. The vertical axis reports cumulative changes in log sectoral employment shares. The shock measure is constructed using changes in external import demand weighted by lagged Peruvian export exposure across destination markets and sectors. Shaded bands denote 95 percent confidence intervals based on standard errors clustered at the sector level. Sectoral employment shares are computed using household survey data from ENAHO aggregated to ISIC Rev. 3 two-digit sectors. The dashed vertical line marks the shock period ($h = 0$), and the dashed horizontal line marks zero change in sectoral employment shares.

6 Conclusion

This paper studies how wealth distribution and financial frictions shape specialization and aggregate dynamics. The central mechanism is a feedback loop between wealth, production, and prices. Because entrepreneurs face collateral constraints, their ability to expand depends on their balance sheets. Wealth accumulation then governs future production, while sectoral output affects prices, which in turn shape profits and savings. This interaction makes the distribution of wealth a state variable that determines both the path of adjustment and the allocation of resources across sectors.

A key implication is that the role of prices differs sharply across environments. In a closed economy, relative prices respond to sectoral expansion and act as a correction mechanism, reducing initial asymmetries and promoting convergence in allocation and productivity. In a small open economy, prices are fixed at world levels, removing this adjustment margin. As a result, differences in initial wealth and within-sector allocation translate into persistent differences in specialization. Trade does not eliminate distortions; instead, it allows them to propagate over time through wealth accumulation.

These results highlight that specialization is not determined solely by fundamentals such as technology or endowments. The distribution of wealth and the presence of financial frictions play a central role in shaping both transition dynamics and long-run outcomes. More broadly, the findings suggest that the gains from trade depend on initial conditions and domestic financial structure, and that policies affecting access to capital can influence not only aggregate growth but also the direction of specialization.

The empirical analysis provides preliminary evidence consistent with the mechanism using sector-level external trade shocks combined with firm-level and sector-level data from Peru. Rather than focusing on a single historical episode, I construct measures of changes in external demand, supply, and international prices using global trade flows excluding Peru. These measures capture variation in external profitability conditions across sectors and can be interpreted as changes in the environment faced by a small open economy.

Combining these sectoral trade shocks with firm-level balance-sheet data, the results show that firm expansion is heterogeneous in ways consistent with the model. More productive firms tend to expand capital and labor more strongly following favorable external demand shocks, suggesting that external conditions affect reallocation partly through firm-level accumulation dynamics. Sector-level local projections also show that external demand shocks are followed by gradual and persistent changes in employment shares, consistent with the idea that trade shocks affect specialization over time rather than only contemporaneous allocation.

These results should be interpreted as preliminary evidence on the mechanism. Their value is to connect the theory to empirical patterns that are central for the dynamic Stolper–Samuelson channel emphasized in the paper: external shocks affect profitability, profitability shapes accumulation, and accumulation gradually changes sectoral composition. A useful next step is to discipline the model quantitatively using these moments, especially the persistence of sectoral responses and the heterogeneous expansion of productive firms. This exercise would also require measuring the parameters governing productivity processes, as well as the initial levels and distributions of capital and productivity across sectors. More broadly, the evidence suggests that persistent specialization patterns in open economies may reflect not only fundamentals or static comparative advantage, but also endogenous wealth accumulation and reallocation dynamics following external shocks.

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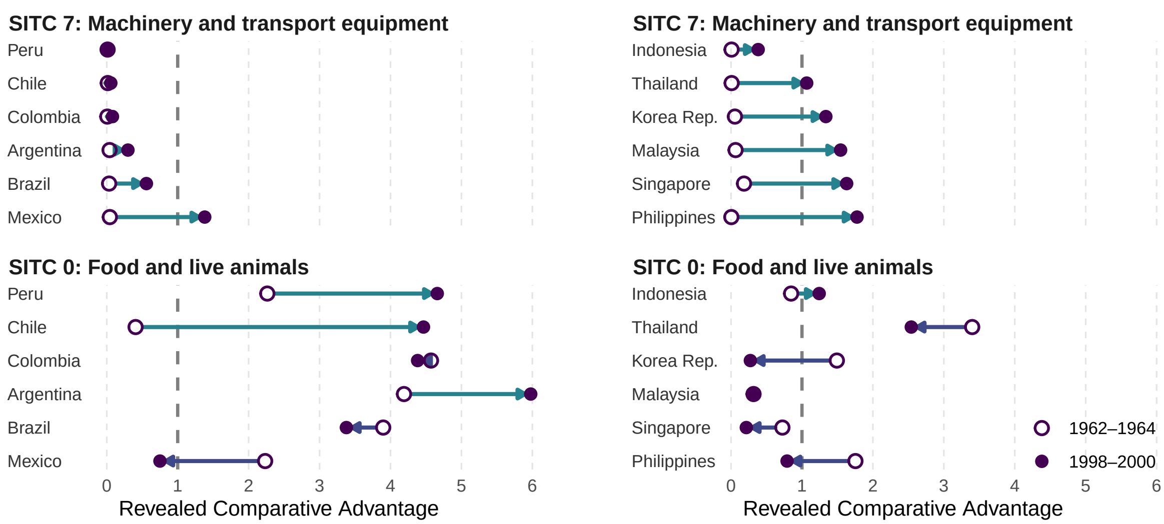
A Cross-Country Evidence on Divergence in Sectoral Specialization

This appendix presents cross-country evidence on the evolution of sectoral specialization that connects the paper with the divergence between East Asia and Latin America (Evans, 1995; Rodrik, 1995; Ocampo, Rada, & Taylor, 2009). Figure A.1 plots changes in revealed comparative advantage (RCA) for a set of Latin American and Asian economies between the early 1960s and the late 1990s.

Two patterns emerge. First, divergence in specialization is concentrated in capital-intensive sectors, such as machinery and transport equipment. Countries with similar initial RCA levels experience markedly different trajectories, with Asian economies moving toward strong comparative advantage and Latin American economies remaining below the threshold. Second, this divergence is not present in less capital-intensive sectors, such as food products, where changes are more muted and do not display a systematic regional pattern.

These patterns suggest that specialization paths are not determined solely by initial conditions or static comparative advantage. Instead, they point to mechanisms linked to capital accumulation and sectoral dynamics that can generate persistent divergence across otherwise similar economies. While this evidence is not used directly in the main text, it provides background motivation for the model's focus on wealth distribution, financial frictions, and dynamic specialization.



(a) Latin America

(b) Asia

Figure A.1: Divergence in Sectoral Specialization between Asian and Latin American Economies

Notes: The figure plots revealed comparative advantage (RCA) following Balassa (1965) for a set of Latin American and Asian economies. RCA is defined as $RCA_{c,i,t} = \frac{X_{c,i,t}/X_{c,t}}{X_{w,i,t}/X_{w,t}}$, where $X_{c,i,t}$ denotes exports of country c in sector i at time t, $X_{c,t}$ total exports of country c, and $X_{w,i,t}$ and $X_{w,t}$ the corresponding world aggregates. Each row corresponds to a country. Hollow circles denote average RCA in 1962–1964, and filled circles denote average RCA in 1998–2000. Arrows connect initial to final values, indicating the direction and magnitude of changes in specialization. The horizontal axis reports RCA levels, and the dashed vertical line marks $RCA = 1$, the threshold for comparative advantage. Trade data are from Feenstra, Lipsey, Deng, Ma, and Mo (2005).

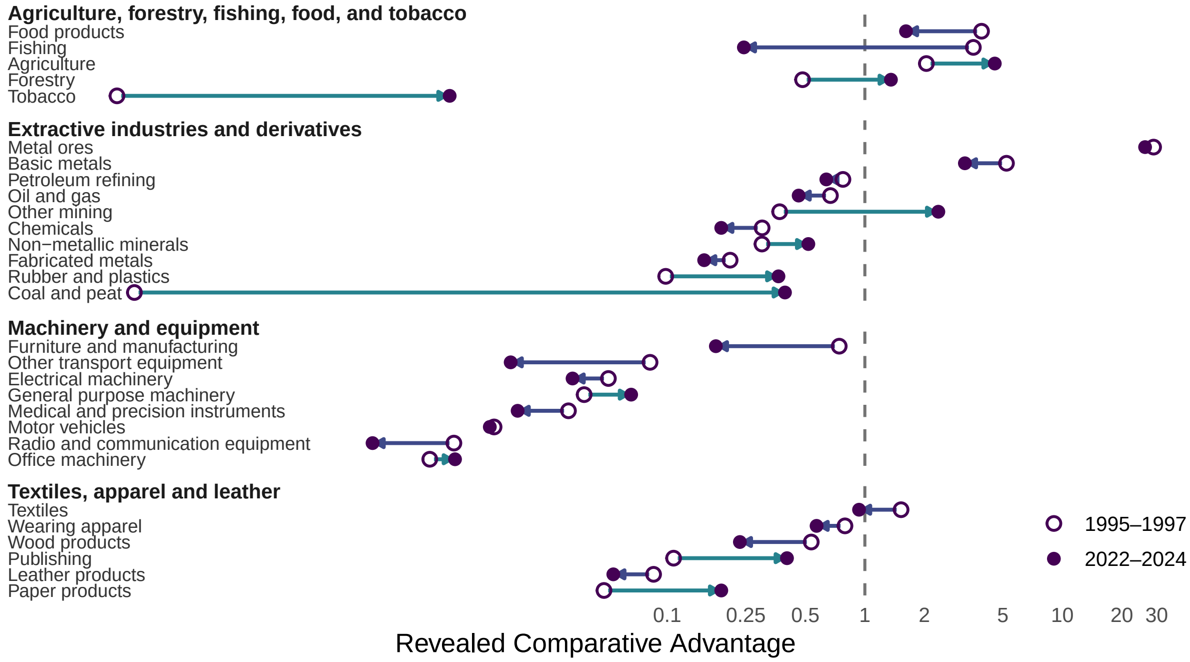


Figure A.2: Direction of Sectoral Specialization in Peru

Notes: The figure plots revealed comparative advantage (RCA) for Peruvian exports across ISIC Rev. 3 two-digit sectors. RCA is defined as $RCA_{i,t} = (X_{Peru,i,t}/X_{Peru,t}) \,/\, (X_{World,i,t}/X_{World,t})$, where $X_{Peru,i,t}$ denotes Peruvian exports in sector i at time t, $X_{Peru,t}$ total Peruvian exports, and $X_{World,i,t}$ and $X_{World,t}$ the corresponding world aggregates. Hollow circles denote average RCA over 1995–1997, and filled circles denote average RCA over 2022–2024. Arrows connect initial to final values, indicating the direction and magnitude of changes in specialization patterns over time. The horizontal axis reports RCA levels on a logarithmic scale, and the dashed vertical line marks $RCA = 1$, the threshold for comparative advantage. Sectors are grouped into broad industry categories to highlight the persistence of specialization patterns across resource-intensive, manufacturing, and machinery-related activities. Trade data are from the BACI International Trade Database harmonized to ISIC Rev. 3 classifications.

B Solution of the Static Problem of the Firm

The entrepreneur's static problem is:

$$\Pi_i(a,z) = \max_{k,\ell} \left\{ p_i (zk)^{\alpha_i} \ell^{1-\alpha_i} - w\ell - (r+\delta)k \ \text{ s.t. } \ k \le \lambda_i a \right\}. \tag{B.1}$$

Step 1: Optimal labor demand. The first-order condition with respect to ℓ is:

$$\frac{\partial \Pi_i}{\partial \ell} = 0 \iff p_i(1-\alpha_i)(zk)^{\alpha_i}\ell^{-\alpha_i} = w \tag{B.2}$$

$$\iff \ell = \left(\frac{1-\alpha_i}{w/p_i}\right)^{\frac{1}{\alpha_i}} zk. \tag{B.3}$$

Step 2: Reduced-form profit function. Substituting optimal labor into profits yields:

$$\Pi_i(a,z) = \max_{k \le \lambda_i a} \left\{ p_i (zk)^{\alpha_i} \ell^{1-\alpha_i} - w\ell - (r+\delta)k \right\} \tag{B.4}$$

$$= \max_{k \le \lambda_i a} \left\{ \left[p_i \left(\frac{1-\alpha_i}{w/p_i}\right)^{\frac{1-\alpha_i}{\alpha_i}} - w \left(\frac{1-\alpha_i}{w/p_i}\right)^{\frac{1}{\alpha_i}} \right] zk - (r+\delta)k \right\}. \tag{B.5}$$

Define

$$\pi_i \equiv p_i \alpha_i \left(\frac{1-\alpha_i}{w/p_i}\right)^{\frac{1-\alpha_i}{\alpha_i}}, \tag{B.6}$$

so that profits can be written as

$$\Pi_i(a,z) = \max_{k \le \lambda_i a} \left\{ \left[\pi_i z - (r+\delta)\right] k \right\}. \tag{B.7}$$

Step 3: Optimal capital choice. Since profits are linear in k, the optimal choice is at a corner:

$$k(a,z) = \begin{cases} \lambda_i a, & \text{if } z \ge \underline{z}_i, \\ 0, & \text{otherwise.} \end{cases} \tag{B.8}$$

The productivity cutoff $\underline{z}_i$ is defined by $\pi_i \underline{z}_i = r + \delta$.

Resulting profit function. Substituting optimal $k(a,z)$ yields:

$$\Pi_i(a,z) = \lambda_i \cdot \max\left\{\pi_i z - (r+\delta),\, 0\right\} a. \tag{B.9}$$

B.1 Solution of the Static Problem of the Firm with Sector-Specific Factors

The entrepreneur's static problem is:

$$\Pi_i(a,z) = \max_{k,\ell,h} \left\{ p_i (zk)^{\alpha_i} \ell^{\beta_i} h^{\gamma_i} - w\ell - q_i h - (r+\delta)k \ \text{ s.t. } \ k \le \lambda_i a \right\}, \tag{B.10}$$

where $\alpha_i, \beta_i, \gamma_i > 0$ and

$$\alpha_i + \beta_i + \gamma_i = 1. \tag{B.11}$$

The variable h denotes a sector-specific factor rented by firms at price q_i. Its aggregate supply is fixed at the sector level and will be imposed in equilibrium.

Step 1: Optimal labor and specific-factor demand. For a given level of capital k, the first-order conditions with respect to ℓ and h are:

$$p_i \beta_i (zk)^{\alpha_i} \ell^{\beta_i - 1} h^{\gamma_i} = w, \tag{B.12}$$

$$p_i \gamma_i (zk)^{\alpha_i} \ell^{\beta_i} h^{\gamma_i - 1} = q_i. \tag{B.13}$$

Dividing the first-order condition for labor by the first-order condition for the specific factor gives

$$h = \frac{\gamma_i}{\beta_i} \frac{w}{q_i} \ell. \tag{B.14}$$

Substituting this expression into the labor first-order condition yields

$$p_i \beta_i z^{\alpha_i} k^{\alpha_i} \left(\frac{\gamma_i}{\beta_i} \frac{w}{q_i} \right)^{\gamma_i} \ell^{\beta_i + \gamma_i - 1} = w. \tag{B.15}$$

Since $\alpha_i + \beta_i + \gamma_i = 1$, this becomes

$$p_i \beta_i z^{\alpha_i} k^{\alpha_i} \left(\frac{\gamma_i}{\beta_i} \frac{w}{q_i} \right)^{\gamma_i} \ell^{-\alpha_i} = w. \tag{B.16}$$

Therefore,

$$\ell_i(k,z) = \left[\frac{p_i \beta_i}{w} \left(\frac{\gamma_i}{\beta_i} \frac{w}{q_i} \right)^{\gamma_i} \right]^{1/\alpha_i} zk. \tag{B.17}$$

Using the input ratio again,

$$h_i(k,z) = \frac{\gamma_i}{\beta_i} \frac{w}{q_i} \left[\frac{p_i \beta_i}{w} \left(\frac{\gamma_i}{\beta_i} \frac{w}{q_i} \right)^{\gamma_i} \right]^{1/\alpha_i} zk. \tag{B.18}$$

Equivalently, the optimal input demands can be written as

$$\ell_i(k,z) = zk \left[\frac{p_i \beta_i}{w} \left(\frac{\gamma_i w}{\beta_i q_i} \right)^{\gamma_i} \right]^{\frac{1}{\alpha_i}}, \tag{B.19}$$

$$h_i(k,z) = zk \left[\frac{p_i \gamma_i}{q_i} \left(\frac{\beta_i q_i}{\gamma_i w} \right)^{\beta_i} \right]^{\frac{1}{\alpha_i}}. \tag{B.20}$$

Thus both non-capital input demands are proportional to zk. Since optimal capital is proportional to wealth whenever the collateral constraint binds, labor and specific-factor demand are also linear in wealth.

Step 2: Reduced-form profit function. From Step 1, optimal non-capital input demands can be written as

$$\ell_i(k,z) = m_{\ell i} zk, \qquad h_i(k,z) = m_{hi} zk, \tag{B.21}$$

where

$$m_{\ell i} \equiv \left[\frac{p_i \beta_i}{w} \left(\frac{\gamma_i w}{\beta_i q_i} \right)^{\gamma_i} \right]^{\frac{1}{\alpha_i}}, \tag{B.22}$$

and

$$m_{hi} \equiv \left[\frac{p_i \gamma_i}{q_i} \left(\frac{\beta_i q_i}{\gamma_i w} \right)^{\beta_i} \right]^{\frac{1}{\alpha_i}}. \tag{B.23}$$

Substituting these expressions into output gives

$$y_i(k,z) = (zk)^{\alpha_i} (m_{\ell i} zk)^{\beta_i} (m_{hi} zk)^{\gamma_i} \tag{B.24}$$

$$= m_{\ell i}^{\beta_i} m_{hi}^{\gamma_i} z^{\alpha_i+\beta_i+\gamma_i} k^{\alpha_i+\beta_i+\gamma_i}. \tag{B.25}$$

Since $\alpha_i + \beta_i + \gamma_i = 1$, this becomes

$$y_i(k,z) = m_{\ell i}^{\beta_i} m_{hi}^{\gamma_i} zk. \tag{B.26}$$

Operating profits after choosing ℓ and h are therefore

$$\max_{\ell,h} \left\{ p_i (zk)^{\alpha_i} \ell^{\beta_i} h^{\gamma_i} - w\ell - q_i h \right\} = \left[p_i m_{\ell i}^{\beta_i} m_{hi}^{\gamma_i} - w m_{\ell i} - q_i m_{hi} \right] zk. \tag{B.27}$$

Using the Cobb-Douglas first-order conditions, payments to labor and the specific factor satisfy

$$w\ell_i = \beta_i p_i y_i, \qquad q_i h_i = \gamma_i p_i y_i. \tag{B.28}$$

Hence operating surplus equals the capital share of revenue:

$$p_i y_i - w\ell_i - q_i h_i = \alpha_i p_i y_i. \tag{B.29}$$

Therefore,

$$\max_{\ell,h} \left\{ p_i (zk)^{\alpha_i} \ell^{\beta_i} h^{\gamma_i} - w\ell - q_i h \right\} = \pi_i z k, \tag{B.30}$$

where

$$\pi_i \equiv \alpha_i p_i m_{\ell i}^{\beta_i} m_{hi}^{\gamma_i}. \tag{B.31}$$

Substituting the expressions for $m_{\ell i}$ and m_{hi}, this can be written as

$$\pi_i \equiv \alpha_i p_i \left(\frac{\beta_i p_i}{w} \right)^{\frac{\beta_i}{\alpha_i}} \left(\frac{\gamma_i p_i}{q_i} \right)^{\frac{\gamma_i}{\alpha_i}}. \tag{B.32}$$

Equivalently, since $\alpha_i + \beta_i + \gamma_i = 1$,

$$\pi_i = \alpha_i p_i^{1/\alpha_i} \left(\frac{\beta_i}{w} \right)^{\beta_i/\alpha_i} \left(\frac{\gamma_i}{q_i} \right)^{\gamma_i/\alpha_i}. \tag{B.33}$$

Thus, the entrepreneur's static problem becomes

$$\Pi_i(a, z) = \max_{k \le \lambda_i a} \left\{ \left[\pi_i z - (r + \delta) \right] k \right\}. \tag{B.34}$$

Step 3: Optimal capital choice. Since profits are linear in k, the optimal choice is at a corner:

$$k_i(a, z) = \begin{cases} \lambda_i a, & \text{if } z \ge \underline{z}_i, \\ 0, & \text{otherwise.} \end{cases} \tag{B.35}$$

The productivity cutoff $\underline{z}_i$ is defined by

$$\pi_i \underline{z}_i = r + \delta. \tag{B.36}$$

Thus,

$$\underline{z}_i = \frac{r + \delta}{\pi_i}. \tag{B.37}$$

Resulting input demands. Substituting the optimal capital choice into the input demands gives:

$$\ell_i(a, z) = \begin{cases} m_{\ell i} z \lambda_i a, & \text{if } z \ge \underline{z}_i, \\ 0, & \text{otherwise,} \end{cases} \tag{B.38}$$

and

$$h_i(a,z) = \begin{cases} m_{hi} z \lambda_i a, & \text{if } z \geq \underline{z}_i, \\ 0, & \text{otherwise.} \end{cases} \tag{B.39}$$

Hence capital, labor, and specific-factor demand remain linear in wealth.

Resulting profit function. Substituting optimal capital demand into the reduced-form problem yields

$$\Pi_i(a,z) = \lambda_i \max\left\{\pi_i z - (r+\delta), 0\right\} a. \tag{B.40}$$

B.1.1 Specific-Factor Market Clearing and Aggregation

Specific-factor market clearing. The sector-specific factor is in fixed supply. In equilibrium, its rental price q_i adjusts so that

$$H_i = \int h_i(a,z) g_i(a,z)\, da\, dz. \tag{B.41}$$

Using the linear policy rules derived above, this condition can be written as

$$H_i = \lambda_i A_i \int_{\underline{z}_i}^{\infty} m_{hi} z o_i(z)\, dz. \tag{B.42}$$

Since m_{hi} depends on the rental price q_i, this condition pins down q_i in equilibrium. As sector i expands, demand for the sector-specific factor rises. Because H_i is fixed, the rental price q_i increases, lowering the operating-surplus coefficient π_i. This creates a congestion force that attenuates complete specialization while preserving the linearity of firm policies in wealth.

Sector-level decreasing returns in scalable inputs. The same aggregation logic used in the baseline model continues to apply. Since individual policies are linear in wealth, sectoral output can be written as

$$Y_i = Z_i K_i^{\alpha_i} L_i^{\beta_i} H_i^{\gamma_i}, \tag{B.43}$$

where

$$\alpha_i + \beta_i + \gamma_i = 1. \tag{B.44}$$

Because H_i is fixed at the sector level, equilibrium output can be interpreted as a decreasing-returns function of the scalable inputs K_i and L_i:

$$Y_i = \widetilde{Z}_i K_i^{\alpha_i} L_i^{\beta_i}, \qquad \widetilde{Z}_i \equiv Z_i H_i^{\gamma_i}. \tag{B.45}$$

Since

$$\alpha_i + \beta_i = 1 - \gamma_i < 1, \tag{B.46}$$

sectoral expansion faces decreasing returns in capital and labor. This provides an additional force limiting complete specialization: as a sector grows, its fixed factor becomes scarcer, the rental price q_i rises, and the sector's profitability advantage declines.

C Solution of the Dynamic Problem of the Entrepreneur

The entrepreneur's dynamic problem

$$\max_{\{c(s)\}_{s\geq t}} \mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)}\ln(c(s))\,ds\right] \quad \text{s.t.} \quad \dot{a}(s) = \Gamma_i(z,s)\,a(s) - P(s)c(s), \tag{C.1}$$

can be represented by the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho V_i(a,z,t) = \max_c \left\{\ln(c) + V_{i,a}(a,z,t)\left[\Gamma_i(z,t)a - P(t)c\right] + V_{i,t}(a,z,t) + \mathcal{L}_z V_i(a,z,t)\right\}, \tag{C.2}$$

where $\mathcal{L}_z$ denotes the infinitesimal generator associated with the productivity process in Equation 4:

$$\mathcal{L}_z V_i(a,z,t) \equiv \mu_i(z)\,V_{i,z}(a,z,t) + \frac{1}{2}\sigma_i(z)^2\,V_{i,zz}(a,z,t). \tag{C.3}$$

This section provides details on the solution of the entrepreneur's dynamic problem.

The first-order and envelope conditions associated with the Hamilton-Jacobi-Bellman equation are:

$$u'(c) = P(t)\,V_{i,a}(a,z,t), \tag{C.4}$$

$$\rho V_{i,a} = V_{i,a}\,\Gamma_i(z,t) + V_{i,at} + \mathcal{L}_z V_{i,a}. \tag{C.5}$$

The following subsections present two alternative derivations of the optimal consumption rule and the induced wealth dynamics. The first uses a Hamilton-Jacobi-Bellman approach with guess and verify, while the second uses a perfect foresight optimal control formulation.

C.1 Hamilton-Jacobi-Bellman with Guess and Verify Approach

Following Moll (2014), consider a guess-and-verify solution that exploits the homothetic structure of the problem. Suppressing sector indices for notational simplicity, the Hamilton–Jacobi–Bellman equation becomes

$$\rho V(a,z,t) = \max_c \left\{\ln c + V_a(a,z,t)\left[\Gamma_i(z,t)a - P(t)c\right] + V_t(a,z,t) + \mathcal{L}_z V(a,z,t)\right\}.$$

The first-order condition with respect to consumption is

$$\frac{1}{c} = P(t)V_a(a,z,t),$$

or equivalently,

$$P(t)c(a,z,t) = \frac{1}{V_a(a,z,t)}.$$

Consider the guess

$$V(a,z,t) = D(z,t) + E(z,t)\log a.$$

Then

$$V_a(a,z,t) = \frac{E(z,t)}{a},$$

implying

$$P(t)c(a,z,t) = \frac{a}{E(z,t)}.$$

Substituting the guess and optimal consumption rule into the HJB equation and collecting terms proportional to $\log a$ yields

$$\rho E(z,t) = 1 + E_t(z,t).$$

A time-invariant solution therefore satisfies

$$E(z) = \frac{1}{\rho}.$$

Hence,

$$P(t)c(a,t) = \rho a(t).$$

Substituting into the wealth accumulation equation yields

$$\dot{a}(t) = [\Gamma_i(z,t) - \rho]\, a(t).$$

C.2 Perfect Foresight Approach

As an alternative, the problem can be solved using a standard optimal control approach. Consider the Hamiltonian:

$$\mathcal{H} = \log c(t) + \mu(t)\left[\Gamma_i(z,t)a(t) - P(t)c(t)\right].$$

The first-order condition with respect to consumption is:

$$\frac{\partial \mathcal{H}}{\partial c(t)} = 0 \quad \Longleftrightarrow \quad \frac{1}{c(t)} = \mu(t)P(t) \quad \Longleftrightarrow \quad \mu(t) = \frac{1}{P(t)c(t)}.$$

The costate equation is:

$$\dot{\mu}(t) = \rho\mu(t) - \frac{\partial \mathcal{H}}{\partial a(t)} = \rho\mu(t) - \mu(t)\Gamma_i(z,t).$$

Substituting for $\mu(t)$ yields:

$$\frac{\dot{(P(t)c(t))}}{P(t)c(t)} = \Gamma_i(z,t) - \rho.$$

Let $m(t) \equiv P(t)c(t)$. The costate equation can be further rewritten as

$$\dot{m}(t) = [\Gamma_i(z,t) - \rho]\, m(t).$$

Together with the law of motion for wealth,

$$\dot{a}(t) = \Gamma_i(z,t)a(t) - m(t),$$

this implies that the ratio $x(t) \equiv m(t)/a(t)$ evolves according to

$$\dot{x}(t) = x(t)\big(x(t) - \rho\big).$$

The optimal path consistent with the transversality condition is the stationary ratio $x(t) = \rho$, so that

$$P(t)c(t) = \rho a(t).$$

Hence,

$$\dot{a}(t) = [\Gamma_i(z,t) - \rho]\, a(t).$$

D Aggregation

- **Law of motion of wealth:**
  - Obtain the sector level wealth growth by integrating the wealth accumulation for all entrepreneurs in that sector:

$$\dot{A}_i(t) = \int \dot{a}(t) dG_i(a, z_i)$$

    Using the savings policy and rewriting in terms of wealth shares:

$$\begin{aligned}\dot{A}_i(t) &= \int s(z_i,t)a(t)dG_i(a,z_i)\\ \dot{A}_i(t) &= A_i(t)\int s(z_i,t)\tfrac{a(t)}{A_i(t)}dG_i(a,z_i)\\ \dot{A}_i(t) &= A_i(t)\int s(z_i,t)o_i(z_i,t)dz_i\end{aligned}$$

  - To obtain the aggregate wealth law of motion, sum up the sectoral wealth laws of motion:

$$\begin{aligned}\dot{A}(t) &= \textstyle\sum_{i\in I}\dot{A}_i(t)\\ \dot{A}(t) &= \textstyle\sum_{i\in I}\int s(z_i,t)o_i(z_i,t)dz_i\\ \dot{A}(t) &= A(t)\textstyle\sum_{i\in I}\int s(z_i,t)\omega_i(z_i,t)dz_i\end{aligned}$$

- **Aggregate output:**
  - Replacing optimal individual capital demand in optimal individual labor demand:

$$\begin{aligned}\ell &= \left(\tfrac{1-\alpha_i}{w/p_i}\right)^{\frac{1}{\alpha_i}} z_i k\\ \ell(a,z_i) &= \begin{cases}\left(\tfrac{\pi_i}{p_i\alpha_i}\right)^{\frac{1}{1-\alpha_i}} z_i\lambda_i a & , z_i \geq \underline{z_i}\\ 0 & , z_i < \underline{z_i}\end{cases}\end{aligned}$$

    with $\pi_i = p_i\alpha_i\left(\frac{1-\alpha_i}{w/p_i}\right)^{\frac{1-\alpha_i}{\alpha_i}}$
  - Replacing $k(a, z_i)$ and $\ell(a, z_i)$ in individual output:

$$\begin{aligned}y_i &= (z_i k)^{\alpha_i}\ell^{1-\alpha_i}\\ y_i(a,z_i) &= \begin{cases} z_i\lambda_i a\tfrac{\pi}{p_i\alpha_i} & , z_i \geq \underline{z_i}\\ 0 & , z_i < \underline{z_i}\end{cases}\end{aligned}$$

- Aggregating output over sector i:

$$
\begin{aligned}
Y_i &= \int_0^\infty \int_0^\infty y_i(a, z_i) g_i(a, z_i)\, da\, dz_i \\
Y_i &= \lambda \frac{\pi_i}{p_i \alpha_i} A_i \int_{\underline{z}_i}^\infty z_i o_i(z_i, t)\, dz_i \\
p_i Y_i &= \left(\frac{\pi_i}{\alpha_i}\right) \lambda X_i A_i
\end{aligned}
$$

with $X_i = \int_{\underline{z}_i}^\infty z_i o_i(z_i, t)\, dz_i$

- Sector level labor demand comes from:

$$
\begin{aligned}
L_i &= \int_0^\infty \int_0^\infty l_i(a, z_i) dG_i(a, z_i) \\
L_i &= \int_0^\infty \int_0^\infty l_i(a, z_i) g_i(a, z_i)\, da\, dz_i \\
L_i &= \int_{\underline{z}_i}^\infty \int_0^\infty \left(\frac{\pi_i}{p_i \alpha_i}\right)^{\frac{1}{1-\alpha_i}} z_i \lambda_i a g_i(a, z_i)\, da\, dz_i \\
L_i &= \int_{\underline{z}_i}^\infty \left(\frac{\pi_i}{p_i \alpha_i}\right)^{\frac{1}{1-\alpha_i}} z_i \lambda_i \int_0^\infty a g_i(a, z_i)\, da\, dz_i \\
L_i &= \int_{\underline{z}_i}^\infty \left(\frac{\pi_i}{p_i \alpha_i}\right)^{\frac{1}{1-\alpha_i}} z_i \lambda_i A_i \left(\int_0^\infty \frac{a g_i(a, z_i)}{A_i}\, da\right) dz_i \\
L_i &= \int_{\underline{z}_i}^\infty \left(\frac{\pi_i}{p_i \alpha_i}\right)^{\frac{1}{1-\alpha_i}} z_i \lambda_i A_i o_i(z_i, t)\, dz_i \\
L_i &= \left(\frac{\pi_i}{p_i \alpha_i}\right)^{\frac{1}{1-\alpha_i}} \lambda_i A_i \int_{\underline{z}_i}^\infty z_i o_i(z_i, t)\, dz_i \\
L_i &= \left(\frac{\pi_i}{p_i \alpha_i}\right)^{\frac{1}{1-\alpha_i}} \lambda_i A_i X_i
\end{aligned}
$$

Notice that $L_i = \left(\frac{\pi_i}{p_i \alpha_i}\right)^{\frac{1}{1-\alpha_i}} \lambda_i A_i X_i \Leftrightarrow \pi_i = p_i \alpha_i \left(\lambda_i X_i\right)^{\alpha_i - 1} A_i^{\alpha_i - 1} L_i^{1 * \alpha_i}$

- Plugging π/α_i in $p_i Y_i$:

$$
\begin{aligned}
p_i Y_i &= \left(p_i \left(\lambda_i X_i\right)^{\alpha_i - 1} A_i^{\alpha_i - 1} L_i^{1 * \alpha_i}\right) \lambda X_i A_i \\
Y_i &= \left(\lambda_i X_i\right)^{\alpha_i} A_i^{\alpha_i} L_i^{1 - \alpha_i}
\end{aligned}
$$

- Sectoral capital demand can be obtained in a similar manner:

$$
\begin{aligned}
K_i &= \int_0^\infty \int_0^\infty k_i(a, z_i) dG_i(a, z_i) \\
K_i &= \int_0^\infty \int_0^\infty k_i(a, z_i) g_i(a, z_i)\, da\, dz_i \\
K_i &= \int_{\underline{z}_i}^\infty \int_0^\infty \lambda_i a_i g_i(a, z_i)\, da\, dz_i \\
K_i &= \lambda_i \int_{\underline{z}_i}^\infty \int_0^\infty a_i g_i(a, z_i)\, da\, dz_i \\
K_i &= \lambda_i A_i (1 - \Omega_i(\underline{z}_i))
\end{aligned}
$$

Also, notice that $K_i = \lambda_i A_i (1 - \Omega_i(\underline{z}_i)) \Leftrightarrow \lambda_i = \frac{\kappa_i}{1 - \Omega_i(\underline{z}_i)}$. Where $\kappa_i \equiv \frac{K_i}{A_i} = \lambda_i \frac{A_i^{\text{effective}}}{A_i}$ is the sector effective capital ratio.

– Plugging λ_i in the previous expression for Y_i:

$$
\begin{aligned}
Y_i &= (\lambda_i X_i)^{\alpha_i} A_i^{\alpha_i} L_i^{1-\alpha_i} \\
Y_i &= \left(\tfrac{\kappa_i X_i}{1-\Omega_i(\underline{z}_i)}\right)^{\alpha_i} A_i^{\alpha_i} L_i^{1-\alpha_i} \\
Y_i &= \left(\tfrac{X_i}{1-\Omega_i(\underline{z}_i)}\right)^{\alpha_i} (\kappa_i A_i)^{\alpha_i} L_i^{1-\alpha_i} \\
Y_i &= \left(\mathbb{E}_{o_i}\left[z_i | z_i \geq \underline{z}_i\right]\right)^{\alpha_i} (\kappa_i A_i)^{\alpha_i} L_i^{1-\alpha_i} \\
Y_i &= Z_i K_i^{\alpha_i} L_i^{1-\alpha_i}
\end{aligned}
$$

where $Z_i = \left(\mathbb{E}_{o_i}\left[z_i | z_i \geq \underline{z}_i\right]\right)^{\alpha_i} = \left(\frac{\int_{\underline{z}_i}^{\infty} z_i o_i(z_i,t)\, dz_i}{1-\Omega_i(\underline{z}_i)}\right)^{\alpha_i}$

– Following a similar process it can be shown that:

$$\dot{A}_i(t) = \alpha_i p_i Y_i + (r-\rho)A_i - (r+\delta)K_i$$

- **Prices:** Using the previous aggregate quantities, it can be shown that factor prices satisfy

$$\frac{w(t)}{p_i(t)} = (1-\alpha_i) Z_i(t) K_i(t)^{\alpha_i} L_i(t)^{-\alpha_i}, \tag{D.1}$$

and

$$r(t) = p_i(t)\alpha_i \zeta_i(t) Z_i(t) K_i(t)^{\alpha_i - 1} L_i(t)^{1-\alpha_i} - \delta, \tag{D.2}$$

where

$$\zeta_i(t) = \frac{\underline{z}_i(t)}{\mathbb{E}_{o_i,t}\left[z \mid z \geq \underline{z}_i\right]} \in [0,1] \tag{D.3}$$

capturing the effect of selection on the return to capital.

E Aggregate Productivity and Decomposition

While aggregation at the sectoral level is tractable given the linearity of individual policies, heterogeneity in production elasticities across sectors precludes representing aggregate output with a single Cobb–Douglas production function. As a result, changes in sectoral composition affect aggregate productivity through their impact on the relative importance of sectors with different technologies. Aggregation therefore follows the Domar–Hulten approach to growth accounting, whereby aggregate productivity growth can be expressed as a Domar-weighted average of sectoral productivity growth (Domar, 1961; Hulten, 1978).

The within–between decomposition is related to the firm dynamics literature, which separates productivity changes into within-unit improvements and reallocation across units (Baily, Hulten, & Campbell, 1992; Foster, Haltiwanger, & Krizan, 2001). In this environment, however, both sectoral productivity and Domar weights are endogenous, as they are jointly determined by wealth dynamics, selection, and equilibrium relative prices.

Proposition 11 (Aggregate Productivity and Sectoral Reallocation)**.** *Let nominal gross output be*

$$P(t)Y(t) \equiv \sum_{i \in I} p_i(t) Y_i(t),$$

and define the Domar weight (expenditure share) of sector i as

$$\psi_i(t) \equiv \frac{p_i(t) Y_i(t)}{P(t)Y(t)}.$$

(i) Aggregate productivity growth. *Can be written as:*

$$\dot{\log Z^A}(t) = \sum_{i \in I} \psi_i(t) \dot{\log Z_i}(t) = \sum_{i \in I} \psi_i(t) \alpha_i \frac{d}{dt} \log \mathbb{E}_{o_i,t}[z \mid z \geq \underline{z}_i(t)].$$

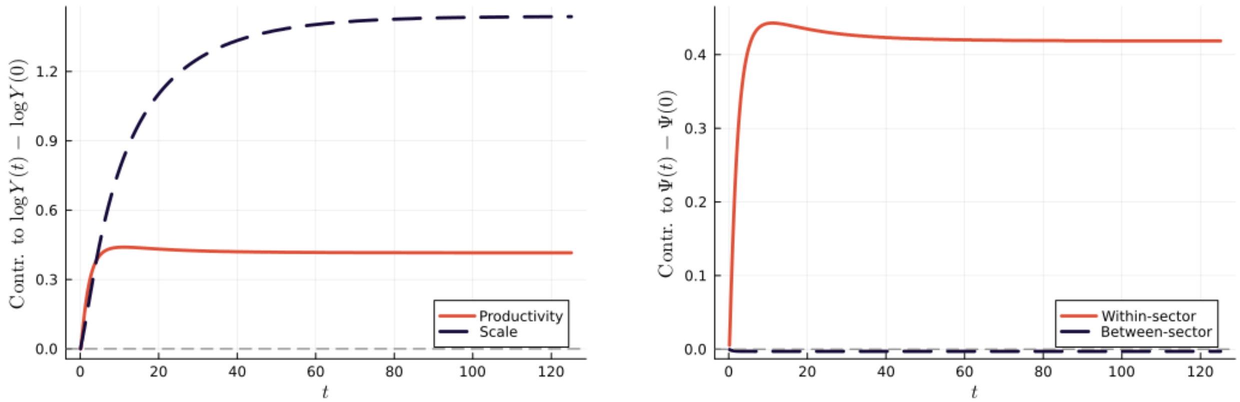
(ii) Within–between decomposition. *The Domar-weighted log productivity index*

$$\Psi(t) \equiv \sum_{i \in I} \psi_i(t) \log Z_i(t)$$

satisfies

$$\dot{\Psi}(t) = \underbrace{\sum_{i \in I} \psi_i(t) \dot{\log Z_i}(t)}_{\text{within-sector}} + \underbrace{\sum_{i \in I} \dot{\psi}_i(t) \log Z_i(t)}_{\text{between-sector}}.$$

This decomposition separates aggregate productivity dynamics into a within-sector component and a between-sector component. The within-sector term reflects changes in productivity driven by selection and wealth dynamics within sectors, while the between-sector term captures reallocation across sectors induced by changes in sectoral profitability and relative prices. Since both $\psi_i(t)$



(a) Growth Decomposition

(b) Cumulative decomposition

Figure E.1: Output, aggregate productivity and sectoral reallocation.

The figure reports aggregate output dynamics and its decomposition in a two-sector closed economy with asymmetric initial wealth–productivity distributions. Panel (a) decomposes output growth, $\log Y(t) - \log Y(0)$, into a productivity component given by the Domar-weighted index $\Psi(t)$ and a residual accumulation/allocation component. Panel (b) decomposes cumulative productivity growth into within-sector and between-sector components, corresponding to $\sum_i \psi_i(t) \log \dot{Z}_i(t)$ and $\sum_i \dot{\psi}_i(t) \log Z_i(t)$, respectively. Aggregate productivity gains are driven primarily by within-sector selection, while the contribution of sectoral reallocation is quantitatively small and short-lived due to the rapid adjustment of relative prices in equilibrium.

and $Z_i(t)$ are endogenous, aggregate productivity dynamics reflect the joint evolution of sectoral composition and within-sector efficiency.

To quantify the role of sectoral reallocation in the closed economy quantitative exercise, Figure E.1 decomposes aggregate productivity dynamics into within-sector and between-sector components using the Domar-based decomposition introduced above.

The within-sector component captures improvements in productivity driven by selection within sectors, while the between-sector component reflects changes in sectoral composition. In the closed economy, the between-sector component tends to decline over time, as relative price adjustments reduce incentives for further reallocation toward initially expanding sectors.

Proof. Proceed in two steps.

Step 1: Aggregate productivity growth. Let nominal gross output be

$$P(t)Y(t) = \sum_{i \in I} p_i(t) Y_i(t),$$

and define Domar weights as

$$\psi_i(t) = \frac{p_i(t) Y_i(t)}{P(t) Y(t)}.$$

Aggregate productivity growth is given by the Divisia index

$$\dot{\log Z^A}(t) = \sum_{i \in I} \psi_i(t) \dot{\log Z_i}(t).$$

Using

$$Z_i(t) = \left(\mathbb{E}_{o_i,t}[z \mid z \geq \underline{z}_i(t)]\right)^{\alpha_i},$$

it follows that

$$\log Z_i(t) = \alpha_i \log \mathbb{E}_{o_i,t}[z \mid z \geq \underline{z}_i(t)],$$

and therefore

$$\dot{\log Z_i}(t) = \alpha_i \frac{d}{dt} \log \mathbb{E}_{o_i,t}[z \mid z \geq \underline{z}_i(t)].$$

Substituting into the Divisia index yields

$$\dot{\log Z^A}(t) = \sum_{i \in I} \psi_i(t) \alpha_i \frac{d}{dt} \log \mathbb{E}_{o_i,t}[z \mid z \geq \underline{z}_i(t)].$$

Step 2: Within–between decomposition. Define the Domar-weighted log productivity index

$$\Psi(t) \equiv \sum_{i \in I} \psi_i(t) \log Z_i(t).$$

Differentiating and applying the product rule,

$$\dot{\Psi}(t) = \sum_{i \in I} \psi_i(t) \dot{\log Z_i}(t) + \sum_{i \in I} \dot{\psi}_i(t) \log Z_i(t).$$

The first term captures within-sector productivity changes, holding expenditure shares fixed. The second term captures reallocation across sectors through changes in expenditure shares. Since $\psi_i(t)$ depends on both sectoral outputs and prices, this term reflects reallocation driven by endogenous changes in quantities and relative prices.

□

F Steady State

A steady state is an equilibrium in which aggregate wealth and its distribution are time-invariant. Formally, for all sectors $i \in I$,

$$\dot{A}_i(t) = 0, \quad g_i(a, z; t) = g_i(a, z), \quad \omega_i(z, t) = \omega_i(z), \quad \text{and} \quad O_i(t) = O_i. \tag{F.1}$$

In this environment, sectoral output is given by

$$Y_i = Z_i K_i^{\alpha_i} L_i^{1-\alpha_i}, \tag{F.2}$$

where Z_i denotes sectoral productivity, which depends on the stationary distribution of active entrepreneurs.

Using the optimality conditions of firms and the definition of profits, capital income in each sector satisfies

$$\alpha_i p_i Y_i = (r + \delta) K_i - (r - \rho) A_i. \tag{F.3}$$

Aggregating across sectors, factor prices are pinned down by market clearing. The wage satisfies

$$w = \frac{1}{\bar{L}} \left(\sum_{i \in I} p_i Y_i - (\rho + \delta) K \right), \tag{F.4}$$

while the interest rate satisfies

$$r = \frac{1}{K} \sum_{i \in I} \zeta_i \left[(r + \delta) K_i - (r - \rho) A_i \right] - \delta. \tag{F.5}$$

In steady state, the objects underlying aggregate productivity admit a simple characterization. Since sectoral productivities and expenditure shares are constant over time, $\dot{\log Z_i}(t) = 0$ and $\dot{D}_i(t) = 0$ for all i, implying zero aggregate productivity growth, $\dot{\log Z^A}(t) = 0$. The Domar weights D_i become time-invariant and reflect the stationary allocation of expenditure across sectors, while sectoral productivity levels Z_i are pinned down by the stationary distribution of entrepreneurs and the corresponding selection thresholds.

As a result, the within–between decomposition reduces to a static accounting identity: aggregate productivity is fully characterized by the cross-sectional distribution of sectoral productivities and their Domar weights, both of which are endogenously determined in equilibrium through wealth dynamics, selection, and relative prices.

G Wealth Dynamics

This appendix derives the evolution of wealth shares used in the main text. The procedure follows three steps. First, it states the general Fokker–Planck equation that governs the evolution of densities associated with Itô diffusion processes. The derivation and intuition behind this equation are standard and can be found in Risken (1996); Gardiner (2009); Pavliotis (2014); Øksendal (2013).

Second, the Fokker–Planck equation is applied to the joint process for productivity and wealth to derive the law of motion for the wealth-share density. Third, these dynamics are decomposed into within-sector and across-sector components, which are used in the analysis in the main text.

G.1 The General Fokker-Planck Equation

Consider a d-dimensional Itô diffusion process $X_t \in \mathbb{R}^d$ governed by:

$$dX_t = \mu(X_t, t)\, dt + \Sigma(X_t, t)\, dW_t,$$

where:

- $\mu : \mathbb{R}^d \times \mathbb{R} \to \mathbb{R}^d$ is the drift vector,
- $\Sigma : \mathbb{R}^d \times \mathbb{R} \to \mathbb{R}^{d\times m}$ is the diffusion matrix,
- W_t is an m-dimensional Wiener process.

Then, the probability density $\phi(x,t)$ of X_t evolves according to the Fokker–Planck equation:

$$\frac{\partial \phi(x,t)}{\partial t} = -\nabla \cdot [\mu(x,t)\phi(x,t)] + \frac{1}{2}\sum_{i,j=1}^{d} \frac{\partial^2}{\partial x_i \partial x_j}\left[D_{ij}(x,t)\phi(x,t)\right],$$

where $D(x,t) = \Sigma(x,t)\Sigma(x,t)^\top$ is the diffusion (covariance) matrix.

G.2 Derivation of the Wealth Share Dynamics

Apply the general Fokker–Planck equation to the joint process $X_t = (z(t), a(t))$, where $\phi(x,t) = g_i(z,a,t)$:

1. Start from the bivariate system implied by: **sectoral productivity dynamics** and the **optimal savings policy** for sector i:

$$\begin{aligned} dz &= \mu_i(z)\, dt + \sigma_i(z)\, dW_t \\ da &= s_i(z,t)\, a\, dt \end{aligned}$$

   Note that wealth follows a deterministic drift (conditional on z), so there is no diffusion term in the a-dimension.

2. Then, $g_i(z, a, t)$ evolves according to the bivariate Fokker–Planck equation:

$$\frac{\partial g_i}{\partial t} = -\frac{\partial}{\partial z}[\mu_i(z)g_i] - \frac{\partial}{\partial a}[s_i(z,t)ag_i] + \frac{1}{2}\frac{\partial^2}{\partial z^2}[\sigma_i^2(z)g_i]$$

Since shocks affect only productivity, there are no cross-partial diffusion terms involving a.

3. Define $\omega_i(z, t)$ as the wealth-share density over productivity z in sector i:

$$\omega_i(z,t) := \frac{1}{A(t)} \int_0^\infty a\, g_i(z,a,t)\, da$$

Then, its time derivative is:

$$\frac{\partial \omega_i(z,t)}{\partial t} = \frac{1}{A(t)} \int_0^\infty a \frac{\partial g_i}{\partial t}\, da - \frac{\dot{A}(t)}{A(t)}\omega_i(z,t)$$

4. Substituting from the Fokker–Planck equation into the expression for $\partial_t \omega_i(z,t)$ yields:

$$\frac{\partial \omega_i(z,t)}{\partial t} = \frac{1}{A(t)} \int_0^\infty a \Bigg\{ -\frac{\partial}{\partial z}\big[\mu_i(z)g_i\big] - \frac{\partial}{\partial a}\big[s_i(z,t)\, a\, g_i\big] + \frac{1}{2}\frac{\partial^2}{\partial z^2}\big[\sigma_i^2(z)g_i\big] \Bigg\} da - \frac{\dot{A}(t)}{A(t)}\omega_i(z,t).$$

Consider the term involving derivatives with respect to a:

$$\int_0^\infty a \frac{\partial}{\partial a}\big[s_i(z,t)\, a\, g_i\big]\, da.$$

Applying integration by parts,

$$\int_0^\infty u\, dv = [uv]_0^\infty - \int_0^\infty v\, du,$$

with $u = a$ and $dv = \frac{\partial}{\partial a}\big[s_i(z,t)\, a\, g_i\big] da$, yields

$$\int_0^\infty a \frac{\partial}{\partial a}\big[s_i(z,t)\, a\, g_i\big]\, da = [a\, s_i(z,t)\, a\, g_i(z,a,t)]_0^\infty - \int_0^\infty s_i(z,t)\, a\, g_i(z,a,t)\, da.$$

Assuming $a^2 g_i(a, z, t) \to 0$ as $a \to 0$ and $a \to \infty$; the boundary term vanishes, so

$$\int_0^\infty a \frac{\partial}{\partial a}\big[s_i(z,t)\, a\, g_i\big]\, da = -\int_0^\infty s_i(z,t)\, a\, g_i(z,a,t)\, da.$$

Similarly, derivatives with respect to z can be taken outside the integral:

$$\int_0^\infty a\, \frac{\partial}{\partial z}\big[\mu_i(z)g_i\big]\, da = \frac{\partial}{\partial z}\left[\mu_i(z) \int_0^\infty a\, g_i\, da\right],$$

and analogously for the second derivative term.

Using the definition of $\omega_i(z,t)$, this implies

$$\frac{\partial \omega_i(z,t)}{\partial t} = \left[s_i(z,t) - \frac{\dot{A}(t)}{A(t)} \right] \omega_i(z,t) - \frac{\partial}{\partial z}\left[\mu_i(z)\omega_i(z,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial z^2}\left[\sigma_i^2(z)\omega_i(z,t)\right]. \tag{G.1}$$

G.3 Decomposition of Wealth Dynamics

This appendix derives the decomposition of the evolution of wealth shares into within- and across-sector components.

Step 1: Evolution of wealth-weighted distribution. Recall that the wealth-weighted density $\omega_i(z,t)$ evolves according to

$$\frac{\partial \omega_i(z,t)}{\partial t} = \left[s_i(z,t) - \frac{\dot{A}(t)}{A(t)} \right] \omega_i(z,t) - \frac{\partial}{\partial z}\left[\mu_i(z)\omega_i(z,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial z^2}\left[\sigma_i^2(z)\omega_i(z,t)\right]. \tag{G.2}$$

Step 2: Decomposition. Using the identity

$$\omega_i(z,t) = O_i(t)\, o_i(z,t), \tag{G.3}$$

its time derivative satisfies

$$\frac{\partial \omega_i(z,t)}{\partial t} = \dot{O}_i(t)\, o_i(z,t) + O_i(t)\, \frac{\partial o_i(z,t)}{\partial t}. \tag{G.4}$$

Substituting (G.3) and (G.4) into (G.2) and dividing by $O_i(t)$ yields

$$\begin{aligned} \frac{\dot{O}_i(t)}{O_i(t)}\, o_i(z,t) + \frac{\partial o_i(z,t)}{\partial t} = &\left[s_i(z,t) - \frac{\dot{A}(t)}{A(t)} \right] o_i(z,t) \\ &- \frac{\partial}{\partial z}\left[\mu_i(z) o_i(z,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial z^2}\left[\sigma_i^2(z) o_i(z,t)\right]. \end{aligned} \tag{G.5}$$

Step 3: Law of motion for sectoral wealth. Integrating (G.2) over z and using that $\int \omega_i(z,t)\, dz = O_i(t)$, we obtain

$$\dot{O}_i(t) = \int s_i(z,t)\, \omega_i(z,t)\, dz - \frac{\dot{A}(t)}{A(t)} O_i(t). \tag{G.6}$$

Using $\omega_i(z,t) = O_i(t) o_i(z,t)$, this becomes

$$\frac{\dot{O}_i(t)}{O_i(t)} = \bar{s}_i(t) - \frac{\dot{A}(t)}{A(t)}, \tag{G.7}$$

where

$$\bar{s}_i(t) \equiv \int s_i(z,t)\, o_i(z,t)\, dz. \tag{G.8}$$

Step 4: Within-sector dynamics. Substituting (G.7) into (G.5) yields

$$\frac{\partial o_i(z,t)}{\partial t} = \left[s_i(z,t) - \bar{s}_i(t)\right] o_i(z,t) - \frac{\partial}{\partial z}\left[\mu_i(z) o_i(z,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial z^2}\left[\sigma_i^2(z) o_i(z,t)\right]. \tag{G.9}$$

Conclusion. The evolution of wealth shares can therefore be decomposed into: (i) within-sector dynamics governed by deviations from the sectoral average savings rate, and (ii) across-sector dynamics governed by differences in average savings rates across sectors.

H Computation of Equilibrium and Dynamics

Equilibrium objects are computed in two steps at each point in time. First, conditional on the current distribution of wealth and productivities, I solve for factor prices and (in the closed economy) goods prices using a fixed-point procedure that enforces market clearing. This step delivers wages, interest rates, prices, and the implied sectoral allocations and wealth dynamics at time t. Second, given these equilibrium objects, I update the distribution of wealth shares by solving the associated partial differential equation using a finite-difference approximation. The following subsections describe each of these steps in detail.

H.1 Equilibrium at point t

H.1.1 Closed Economy

Given sectoral wealth $\{A_i(t)\}$ and within-sector wealth distributions $\{o_i(z,t)\}$, the task is to find prices $\{p_i(t)\}_{i\in I}$, wage $w(t)$, and interest rate $r(t)$ such that all markets clear.

Prices are normalized so that the CES price index satisfies $P(t) = 1$.

The equilibrium is computed as a fixed point using the following steps:

1. **Initial guesses.** Guess $(w^0, r^0, \{p_i^0\})$, and normalize prices so that $P^0 = 1$.

2. **Static firm block.** Given prices, compute:

    - Profits:
    $$\pi_i^0 = p_i^0 \alpha_i \left(\frac{1-\alpha_i}{w^0/p_i^0} \right)^{\frac{1-\alpha_i}{\alpha_i}}$$
    - Productivity cutoffs:
    $$\underline{z}_i^0 = \frac{r^0 + \delta}{\pi_i^0}$$
    - Active-productivity integral:
    $$X_i^0 = \int_{\underline{z}_i^0}^{\infty} z\, o_i(z,t)\, dz$$
    - Active mass:
    $$1 - \Omega_i(\underline{z}_i^0)$$
    - Sectoral productivity:
    $$Z_i^0 = \left(\frac{X_i^0}{1 - \Omega_i(\underline{z}_i^0)} \right)^{\alpha_i}$$
    - Selection wedge:
    $$\zeta_i^0 = \frac{\underline{z}_i^0}{(Z_i^0)^{1/\alpha_i}}$$

3. **Factor demands and output.**

   - Labor demand:
$$L_i^0 = \lambda_i A_i(t)\, X_i^0 \left(\frac{\pi_i^0}{p_i^0 \alpha_i}\right)^{\frac{1}{1-\alpha_i}}$$
   - Capital demand:
$$K_i^0 = \lambda_i A_i(t)\, [1 - \Omega_i(\underline{z}_i^0)]$$
   - Output:
$$Y_i^0 = Z_i^0 (K_i^0)^{\alpha_i} (L_i^0)^{1-\alpha_i}$$

4. **Wealth accumulation and absorption.**

   - Sectoral wealth dynamics:
$$\dot{A}_i^0 = \alpha_i p_i^0 Y_i^0 + (r^0 - \rho) A_i(t) - (r^0 + \delta) K_i^0$$
   - Total absorption:
$$P^0 D^0 = w^0 \bar{L} + \rho A(t) + \dot{A}(t) + \delta A(t)$$
   - Sectoral demand:
$$D_i^0 = \upsilon_i \left(\frac{p_i^0}{P^0}\right)^{-\eta} D^0$$

5. **Excess demands.**

   - Goods:
$$G_i^0 = Y_i^0 - D_i^0$$
   - Labor:
$$L_{\text{diff}}^0 = \frac{\sum_i L_i^0 - \bar{L}}{\bar{L}}$$
   - Capital:
$$K_{\text{diff}}^0 = \frac{\sum_i K_i^0 - A(t)}{A(t)}$$

6. **Update prices.**

   - Wage (labor clearing):
$$w^1 = w^0 \left(1 + \gamma_w L_{\text{diff}}^0\right)$$
   - Interest rate (capital clearing):
$$r^1 = r^0 + \gamma_r K_{\text{diff}}^0$$

- Goods prices (goods clearing):

$$p_i^1 = p_i^0 \left(1 - \gamma_p \frac{G_i^0}{Y_i^0}\right)$$

- Normalize prices so that $P^1 = 1$

7. **Iteration.** Repeat until convergence:

$$\max\left\{|L_{\text{diff}}|, |K_{\text{diff}}|, \max_i |G_i/Y_i|\right\} < \varepsilon$$

H.1.2 Small Open Economy with Financial Autarky

Given sectoral wealth $\{A_i(t)\}$, within-sector wealth distributions $\{o_i(z,t)\}$, and exogenous goods prices $\{p_i^*\}_{i\in I}$, the task is to find the wage $w(t)$ and interest rate $r(t)$ such that factor markets clear.

Since goods prices are fixed, goods market clearing does not pin down prices, and the equilibrium reduces to clearing labor and capital markets given sectoral profitability.

The equilibrium is computed as follows:

1. **Initial guesses.** Guess (w^0, r^0).

2. **Static firm block.** Given (w^0, r^0) and world prices $\{p_i^*\}$, compute:

   - Profits:
   $$\pi_i^0 = p_i^* \alpha_i \left(\frac{1-\alpha_i}{w^0/p_i^*}\right)^{\frac{1-\alpha_i}{\alpha_i}}$$
   - Productivity cutoffs:
   $$\underline{z}_i^0 = \frac{r^0 + \delta}{\pi_i^0}$$
   - Active-productivity integral:
   $$X_i^0 = \int_{\underline{z}_i^0}^{\infty} z\, o_i(z,t)\, dz$$
   - Active mass:
   $$1 - \Omega_i(\underline{z}_i^0)$$
   - Sectoral productivity:
   $$Z_i^0 = \left(\frac{X_i^0}{1 - \Omega_i(\underline{z}_i^0)}\right)^{\alpha_i}$$
   - Selection wedge:
   $$\zeta_i^0 = \frac{\underline{z}_i^0}{(Z_i^0)^{1/\alpha_i}}$$

3. **Factor demands and output.**

   - Labor demand:
$$L_i^0 = \lambda_i A_i(t)\, X_i^0 \left(\frac{\pi_i^0}{p_i^* \alpha_i}\right)^{\frac{1}{1-\alpha_i}}$$
   - Capital demand:
$$K_i^0 = \lambda_i A_i(t)\,[1 - \Omega_i(\underline{z}_i^0)]$$
   - Output:
$$Y_i^0 = Z_i^0 (K_i^0)^{\alpha_i} (L_i^0)^{1-\alpha_i}$$

4. **Factor market residuals.**

   - Labor:
$$L_{\text{diff}}^0 = \frac{\sum_i L_i^0 - \bar{L}}{\bar{L}}$$
   - Capital:
$$K_{\text{diff}}^0 = \frac{\sum_i K_i^0 - A(t)}{A(t)}$$

5. **Update factor prices.**

   - Wage:
$$w^1 = w^0 \left(1 + \gamma_w L_{\text{diff}}^0\right)$$
   - Interest rate:
$$r^1 = r^0 + \gamma_r K_{\text{diff}}^0$$

6. **Iteration.** Repeat until convergence:
$$\max\{|L_{\text{diff}}|, |K_{\text{diff}}|\} < \varepsilon$$

7. **Post-solution objects.**

   Given equilibrium (w, r), compute:

   - Sectoral wealth dynamics:
$$\dot{A}_i(t) = \alpha_i p_i^* Y_i(t) + (r(t) - \rho) A_i(t) - (r(t) + \delta) K_i(t)$$
   - Total absorption:
$$P^*(t) D(t) = w(t)\bar{L} + \rho A(t) + \dot{A}(t) + \delta A(t)$$

- Sectoral absorption:

$$D_i(t) = \upsilon_i \left(\frac{p_i^*}{P^*(t)} \right)^{-\eta} D(t)$$

- Net exports (residual):

$$NX_i(t) = Y_i(t) - D_i(t)$$

H.2 Solving the PDE via Approximation

1. Write equation G.1 as a generic second order PDE:

$$\frac{\partial \omega_i(z,t)}{\partial t} = u_i(z,t)\omega_i(z,t) + v_i(z)\frac{\partial \omega_i(z,t)}{\partial z} + x_i(z)\frac{\partial^2 \omega_i(z,t)}{\partial z^2} \tag{H.1}$$

2. Apply forward-difference approximation in time dimension and central-difference approximations in the productivity domain dimension:

$$\begin{aligned}
\frac{\partial \omega_i(z_k,t^n)}{\partial t} &\approx \frac{\omega_i(z_k,t^{n+1}) - \omega_i(z_k,t^n)}{\Delta t} \\
\frac{\partial \omega_i(z_k,t^n)}{\partial z} &\approx \frac{\omega_i(z_{k+1},t^n) - \omega_i(z_{k-1},t^n)}{2\Delta z} \\
\frac{\partial^2 \omega_i(z_k,t^n)}{\partial z^2} &\approx \frac{\omega_i(z_{k+1},t^n) - 2\omega_i(z_k,t^n) + \omega_i(z_{k-1},t^n)}{(\Delta z)^2}
\end{aligned}$$

   where $\{z_k\}_{k=1}^K$ are the elements of the discrete grid for z and (t^n) is the discretized time sequence such that $t^{n+1} = t^n + \Delta t$.

3. Using these to approximate (H.1) in an implicit scheme (right-hand side evaluated in $n+1$):

$$\begin{aligned}
\frac{\omega_i(z_k,t^{n+1}) - \omega_i(z_k,t^n)}{\Delta t} = \; & u_i(z_k,t)\omega_i(z,t^n) \\
& +v_i(z_k)\frac{\omega_i(z_{k+1},t^{n+1}) - \omega_i(z_{k-1},t^{n+1})}{2\Delta z} \\
& +x_i(z_k)\frac{\omega_i(z_{k+1},t^{n+1}) - 2\omega_i(z_k,t^{n+1}) + \omega_i(z_{k-1},t^{n+1})}{(\Delta z)^2}
\end{aligned} \tag{H.2}$$

   Rearranging:

$$\omega_i(z_{k-1},t^{n+1})f_i(z_k) + \omega_i(z_k,t^{n+1})g_i(z_k) + \omega_i(z_{k+1},t^{n+1})h_i(z_k) = \omega_i(z_k,t^n) \tag{H.3}$$

   Where $f_i(z_k)$, $g_i(z_k)$, and $h_i(z_k)$ are the following functions of $a_i(z_k,t^n)$, $b_i(z_k)$, $c_i(z_k)$, Δt, and

Δz:

$$
\begin{aligned}
f_i(z_k) &= v_i(z_k)\frac{\Delta t}{2\Delta z} - x_i(z_k)\frac{\Delta t}{(\Delta z)^2} \\
g_i(z_k) &= 1 - u_i(z_k)\Delta t + 2x_i(z_k)\frac{\Delta t}{(\Delta z)^2} \\
h_i(z_k) &= -v_i(z_k)\frac{\Delta t}{2\Delta z} - x_i(z_k)\frac{\Delta t}{(\Delta z)^2}
\end{aligned}
$$

4. With boundary conditions $\omega_i(z_1, t^{n+1}) = 0$ and $\Delta z \sum_{k=1}^{K} \omega_i(z_k, t^{n+1}) = O_i(t^{n+1})$. The problem now becomes:

$$
\begin{bmatrix}
g_i(z_2) & h_i(z_2) & 0 & 0 & \dots & 0 \\
f_i(z_3) & g_i(z_3) & h_i(z_3) & 0 & \dots & 0 \\
0 & f_i(z_4) & g_i(z_4) & h_i(z_4) & \dots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\
\dots & \dots & f_i(z_{K-2}) & g_i(z_{K-2}) & h_i(z_{K-2}) & 0 \\
\dots & \dots & \dots & f_i(z_{K-1}) & g_i(z_{K-1}) & h_i(z_{K-1}) \\
\Delta z & \Delta z & \dots & \dots & \Delta z & \Delta z
\end{bmatrix}
\begin{bmatrix}
\omega_i(z_2, t^{n+1}) \\
\omega_i(z_3, t^{n+1}) \\
\omega_i(z_4, t^{n+1}) \\
\vdots \\
\omega_i(z_{K-2}, t^{n+1}) \\
\omega_i(z_{K-1}, t^{n+1}) \\
\omega_i(z_K, t^{n+1})
\end{bmatrix}
=
\begin{bmatrix}
\omega_i(z_2, t^{n}) \\
\omega_i(z_3, t^{n}) \\
\omega_i(z_4, t^{n}) \\
\vdots \\
\omega_i(z_{K-2}, t^{n}) \\
\omega_i(z_{K-1}, t^{n}) \\
O_i(t^{n+1})
\end{bmatrix}
$$

Which can be solved for $\omega_i(t^{n+1})$ by inverting the matrix in the left.

H.2.1 Specific Solution

From the OU Process for the log of productivity:

$$
dz_i = \frac{1}{\theta_i}\left(-\log z_i + \frac{\sigma_i^2}{2}\right) z_i\, dt + \sigma_i \sqrt{1/\theta_i} z_i\, dW
$$

Plugging $\mu_i(z_i) = \frac{1}{\theta_i}\left(-\log z_i + \frac{\sigma_i^2}{2}\right) z_i$ and $\sigma_i(z_i) = \sigma_i \sqrt{1/\theta_i} z_i$, the wealth share dynamics equation becomes:

$$
\frac{\partial \omega_i(z,t)}{\partial t} = \left[s_i(z,t) - \frac{\dot{A}(t)}{A(t)} + \frac{1}{\theta_i}\left(1 + \log z + \frac{\sigma_i^2}{2}\right)\right] \omega_i(z,t) + \frac{1}{\theta_i}\left(\log z + \frac{3}{2}\sigma_i^2\right) z \frac{\partial \omega_i(z,t)}{\partial t} + \frac{\sigma_i^2}{2\theta_i}\frac{\partial^2 \omega_i(z,t)}{\partial z^2}
$$

That is: $u_i(z,t) = s_i(z,t) - \frac{\dot{A}(t)}{A(t)} + \frac{1}{\theta_i}\left(1 + \log z + \frac{\sigma_i^2}{2}\right)$, $v_i(z,t) = \frac{1}{\theta_i}\left(\log z + \frac{3}{2}\sigma_i^2\right) z$, and $x_i(z,t) = \frac{\sigma_i^2}{2\theta_i}$

I Parametrization and Numerical Implementation

Table I.1: Baseline Parameters for Closed-Economy Simulations

Parameter / Object	Description	Baseline value
Panel A: Environment		
I	Number of sectors	2
$\bar{L}$	Aggregate labor endowment	1.0
Panel B: Preferences, Technology, and Constraints		
η	Elasticity of substitution across sectoral goods	2.0
υ_i	CES weights in final-good aggregator	$1/I$
ρ	Discount rate	0.05
δ	Depreciation rate	0.05
α_i	Capital share in sector i	0.50 (each sector)
λ_i	Collateral parameter in sector i	1.20 (each sector)
Panel C: Productivity Process and Initial Conditions		
θ_i	Mean-reversion parameter of log productivity	6.25 (each sector)
σ_i	Volatility of log productivity	0.65 (each sector)
$\text{Corr}_i = e^{-1/\theta_i}$	One-period persistence	Implied
$\text{std}_i = \sigma_i \sqrt{\theta_i/2}$	Stationary std. dev. of log productivity	Implied
$\rho_{a,z,i}(0)$	Initial correlation between wealth and productivity	0.50 (each sector)
$\mu_{\log z,i,0}$	Initial mean of log productivity	0.0
$\sigma_{\log z,i,0}$	Initial std. dev. of log productivity	std_i
$\mu_{\log a,i,0}$	Initial mean of log wealth	0.0
$\sigma_{\log a,i,0}$	Initial std. dev. of log wealth	std_i
$A_{i,0}$	Initial wealth in sector i	Implied by initialization
Panel D: Computational Parameters		
$z_{\min}$	Lower bound of productivity grid	10^{-4}
$z_{\max}$	Upper bound of productivity grid	$Q_{0.99}(\log \mathcal{N}(0, \max_i \text{std}_i))$
N_z	Number of productivity grid points	1000
T	Simulation horizon	125
N_t	Number of time steps	1250
Δt	Time step size	$T/(N_t - 1)$
$p_i(0)$	Initial guess for sectoral prices	1.0
$w(0)$	Initial wage guess	1.0
$r(0)$	Initial interest rate guess	0.03

Notes: Parameters follow standard values, and the productivity process generates persistent and dispersed firm-level heterogeneity. The one-sector and two-sector economies share the same aggregate wealth and marginal distributions, with asymmetry arising only from differences in the correlation between wealth and productivity across sectors.

The baseline parameterization is chosen to provide a transparent and symmetric benchmark for the quantitative exercises. Preference and technology parameters follow standard values in the macroeconomic literature. The parameters governing the productivity process are selected to generate persistent and dispersed productivity across entrepreneurs, consistent with empirical evidence on firm-level dynamics.

Initial conditions are constructed using sector-specific joint distributions over wealth and productivity. For each sector i, I assume a lognormal specification for (a, z) characterized by $(\mu_{\log a,i,0}, \sigma_{\log a,i,0}, \mu_{\log z,i,0}, \sigma_{\log z,i,0})$ and a correlation parameter $\rho_{a,z,i}(0)$. This implies sector-specific wealth distributions and initial allocations of wealth across productivity levels.

To ensure comparability across specifications, I normalize initial wealth so that total wealth is fixed at a common value $A(0)$ and sectoral wealth shares are identical across sectors in the baseline. In the one-sector benchmark, I aggregate sectoral distributions so that both total initial wealth and the wealth-weighted productivity distribution coincide with those in the multisector economy. In the two-sector economy, asymmetry is introduced through differences in the correlation between wealth and productivity across sectors, holding marginal distributions and aggregate scale fixed.

J Persistent Specialization in a Small Open Economy: A Sector-Specific Capital Benchmark

The purpose of this appendix is to provide a stripped-down example of the mechanism studied in the paper. The model has two sectors, mobile labor, fixed world prices, and sector-specific capital accumulation. The goal is to derive a closed-form characterization of how the interaction between capital accumulation and fixed world prices can generate persistent specialization patterns.

The appendix considers two benchmark exercises. First, if productivity differences across sectors are permanent, the economy converges to full specialization in the more productive sector. Second, if productivity differences are only temporary, aggregate capital eventually converges to the standard Solow steady state once technologies become identical again. However, the sectoral composition of capital continues to reflect the history of those temporary productivity differences. As a result, temporary productivity advantages can generate persistent differences in sectoral capital and output even after productivity differences disappear.

Importantly, the appendix shows that persistent specialization patterns can arise even without heterogeneous entrepreneurs, financial frictions, or a CES demand system. The key "trick" is that labor is mobile across sectors, while capital is accumulated within sectors. Mobile labor links the two sectors in general equilibrium. However, because the economy is small and open, goods prices are fixed at world levels. As a result, sectoral expansion does not lower the relative price of the expanding sector. This removes the closed-economy force that would otherwise reduce initial asymmetries.

The appendix therefore should not be interpreted as a literal reduced-form version of the main model. In particular, capital in this environment is accumulated within sectors, rather than being allocated through an integrated economy-wide financial market as in the heterogeneous-agent framework studied in the paper. As a result, persistence here operates through sectoral accumulation dynamics rather than through the endogenous wealth reallocation mechanism emphasized in the main model.

The purpose of the exercise is instead to isolate, in a transparent environment, the role of fixed world prices in shaping long-run sectoral dynamics. By shutting down the closed-economy relative price adjustment mechanism, temporary asymmetries in productivity translate into persistent differences in sectoral capital stocks and production patterns. The appendix therefore provides an illustrative benchmark for the broader mechanism emphasized in the main text, where similar forces operate through endogenous wealth accumulation and financial frictions in an integrated multisector economy.

J.1 Setup

Environment. Time is continuous. There are two sectors indexed by $i \in \{1, 2\}$. Sector i produces using capital $K_i(t)$ and labor $L_i(t)$ according to

$$Y_i(t) = A_i(t) K_i(t)^\alpha L_i(t)^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{J.1}$$

Total labor supply is fixed and normalized to one:

$$\sum_{i \in \{1,2\}} L_i(t) = 1. \tag{J.2}$$

Labor is perfectly mobile across sectors. Goods prices are fixed at world levels. Without loss of generality, prices can be normalized to one so that all quantities are expressed in real terms:

$$p_i^* = 1, \qquad i \in \{1, 2\}.$$

Capital is accumulated by sector-specific entrepreneurs. To keep the model as close as possible to the Solow model, assume that a constant fraction $s_i \in (0, 1)$ of sectoral output is invested in the same sector. Capital depreciates at rate $\delta > 0$. Thus,

$$\dot{K}_i(t) = s_i Y_i(t) - \delta K_i(t). \tag{J.3}$$

Labor Allocation. At each point in time, labor is allocated competitively across sectors. Hence, the wage equals the value marginal product of labor in every active sector:

$$w(t) = (1 - \alpha) A_i(t) K_i(t)^\alpha L_i(t)^{-\alpha}. \tag{J.4}$$

For any two active sectors, this implies

$$(1 - \alpha) A_1(t) K_1(t)^\alpha L_1(t)^{-\alpha} = (1 - \alpha) A_2(t) K_2(t)^\alpha L_2(t)^{-\alpha}. \tag{J.5}$$

Canceling common terms and rearranging gives

$$\frac{L_1(t)}{L_2(t)} = \left(\frac{A_1(t)}{A_2(t)} \right)^{1/\alpha} \frac{K_1(t)}{K_2(t)}. \tag{J.6}$$

Using $\sum_{i \in \{1,2\}} L_i(t) = 1$, sectoral labor allocations are

$$L_i(t) = \frac{A_i(t)^{1/\alpha} K_i(t)}{\sum_{j \in \{1,2\}} A_j(t)^{1/\alpha} K_j(t)}. \tag{J.7}$$

Labor flows toward sectors with higher productivity and larger capital stocks.

Reduced-Form Sectoral Output. Substituting the labor allocation into production gives

$$Y_i(t) = A_i(t)K_i(t)^{\alpha}\left[\frac{A_i(t)^{1/\alpha}K_i(t)}{\sum_{j\in\{1,2\}} A_j(t)^{1/\alpha}K_j(t)}\right]^{1-\alpha}. \tag{J.8}$$

This simplifies to

$$Y_i(t) = \frac{A_i(t)^{1/\alpha}K_i(t)}{\left[\sum_{j\in\{1,2\}} A_j(t)^{1/\alpha}K_j(t)\right]^{1-\alpha}}. \tag{J.9}$$

Define aggregate effective capital as

$$X(t) \equiv \sum_{i\in\{1,2\}} A_i(t)^{1/\alpha}K_i(t). \tag{J.10}$$

Then sectoral output can be written compactly as

$$Y_i(t) = A_i(t)^{1/\alpha}X(t)^{-(1-\alpha)}K_i(t). \tag{J.11}$$

Capital Accumulation. The law of motion for sectoral capital becomes

$$\dot{K}_i(t) = s_i A_i(t)^{1/\alpha}X(t)^{-(1-\alpha)}K_i(t) - \delta K_i(t). \tag{J.12}$$

Therefore, the growth rate of sectoral capital is

$$g_{K,i} = \frac{\dot{K}_i(t)}{K_i(t)} = s_i A_i(t)^{1/\alpha}X(t)^{-(1-\alpha)} - \delta. \tag{J.13}$$

Taking the difference across sectors gives the law of motion for relative capital:

$$\frac{d}{dt}\log\left(\frac{K_1(t)}{K_2(t)}\right) = X(t)^{-(1-\alpha)}\left[s_1 A_1(t)^{1/\alpha} - s_2 A_2(t)^{1/\alpha}\right]. \tag{J.14}$$

This equation summarizes the mechanism. Relative sectoral capital grows toward the sector with higher productivity or a higher reinvestment rate.

J.2 A Persistent Productivity Gap

First, this toy model shows that if a productivity gap persists across sectors, the economy eventually specializes in the more productive sector.

Suppose both sectors begin with positive capital stocks,

$$K_1(0) > 0, \qquad K_2(0) > 0,$$

and have the same investment rate,

$$s_1 = s_2 = s.$$

Let sector 1 be permanently more productive:

$$A_1(t) = A, \qquad A_2(t) = (1-\tau)A, \qquad 0 < \tau < 1.$$

Using the relative capital growth derived above (Equation J.14),

$$\frac{d}{dt}\log\left(\frac{K_1(t)}{K_2(t)}\right) = sX(t)^{-(1-\alpha)}A^{1/\alpha}\left[1-(1-\tau)^{1/\alpha}\right] > 0. \tag{J.15}$$

Hence, sector 1 accumulates capital faster than sector 2, and its relative size increases over time.

In this case, the economy converges to a corner allocation in which only the more productive sector remains active:

$$K_1^* > 0, \qquad K_2^* = 0. \tag{J.16}$$

At the corner, aggregate capital is equal to capital in sector 1:

$$K^* = K_1^*. \tag{J.17}$$

Since sector 1 has productivity A, aggregate capital satisfies the standard Solow steady-state condition:

$$0 = sA(K^*)^\alpha - \delta K^*. \tag{J.18}$$

Thus,

$$K^* = K_1^* = \left(\frac{sA}{\delta}\right)^{\frac{1}{1-\alpha}}, \qquad K_2^* = 0. \tag{J.19}$$

This benchmark shows that with a permanent productivity gap, fixed world prices imply full specialization in the more productive sector. Aggregate capital converges to the standard Solow steady state, but all capital is eventually concentrated in the more productive sector.

J.3 A Temporary Productivity Gap

The main model shows that under identical fundamentals, the self-financing mechanism tends to reduce initial productivity differences across sectors over time. To capture this idea in the toy model, now suppose that the productivity gap between sectors disappears after a finite period of time.

Suppose the two sectors begin with the same capital stock:

$$K_1(0) = K_2(0) = K_0. \tag{J.20}$$

Assume equal investment rates:

$$s_1 = s_2 = s. \tag{J.21}$$

As before, let sector 1 have productivity

$$A_1(t) = A,$$

while sector 2 has a temporary productivity disadvantage:

$$A_2(t) = \begin{cases} (1-\tau)A, & t < T, \\ A, & t \geq T, \end{cases}, \qquad \text{with } 0 < \tau < 1 \text{ and finite } T. \tag{J.22}$$

Then, for $t < T$, the same logic from the previous subsection implies

$$\frac{d}{dt} \log \left(\frac{K_1(t)}{K_2(t)} \right) > 0. \tag{J.23}$$

Hence, while the productivity gap persists, sector 1 accumulates capital faster than sector 2. Now suppose that at time T, the productivity gap disappears:

$$A_1(t) = A_2(t) = A, \qquad t \geq T. \tag{J.24}$$

From that moment onward, both sectors grow at the same rate as the factor $\left[1 - (1-\tau)^{1/\alpha}\right]$ goes to zero:

$$\frac{d}{dt} \log \left(\frac{K_1(t)}{K_2(t)} \right) = 0 \qquad \Leftrightarrow \qquad g_{K,1} = g_{K,2}. \tag{J.25}$$

Therefore, the relative capital allocation reached at time T persists:

$$R_T \equiv \frac{K_1(t)}{K_2(t)} = \frac{K_1(T)}{K_2(T)} > 1 \qquad \text{for all } t \geq T. \tag{J.26}$$

Even though productivity differences disappear in finite time, the asymmetry generated during the transition remains permanently embedded in the sectoral composition of capital.

Steady State. After time T, both sectors have the same productivity A and the same investment rate s. Therefore, aggregate capital evolves according to:

$$\dot{K}(t) = sAK(t)^\alpha - \delta K(t), \tag{J.27}$$

which is exactly the same law of motion as in the standard Solow model.[5]

Hence, aggregate capital converges again to the standard Solow steady state:

$$K^* = \left(\frac{sA}{\delta}\right)^{\frac{1}{1-\alpha}} \qquad \Leftrightarrow \qquad 0 = sA(K^*)^\alpha - \delta K^*. \tag{J.28}$$

However, the sectoral composition of capital depends on the history of temporary productivity differences. Since relative capital remains constant after time T,

$$\frac{K_1(t)}{K_2(t)} = R_T \qquad \text{for all } t \geq T, \tag{J.29}$$

the steady-state sectoral capital stocks must satisfy

$$K_1^* = \frac{R_T}{1+R_T}K^*, \qquad K_2^* = \frac{1}{1+R_T}K^*, \tag{J.30}$$

with

$$\frac{K_1^*}{K_2^*} = R_T > 1. \tag{J.31}$$

This shows that even though both sectors eventually share identical technologies, the long-run allocation of capital depends on the history of temporary productivity differences. Aggregate capital converges to the standard Solow steady state, but the sectoral composition of that steady

[5] After time T, both sectors share the same productivity level A, so

$$X(t) = A^{1/\alpha} \sum_{i\in\{1,2\}} K_i(t).$$

The sectoral capital accumulation equation becomes

$$g_{K,i} = sA^{1/\alpha}\left[A^{1/\alpha}\sum_{j\in\{1,2\}} K_j(t)\right]^{-(1-\alpha)} - \delta.$$

Using

$$\left[A^{1/\alpha}\right]^{-(1-\alpha)} = A^{-(1-\alpha)/\alpha},$$

and defining aggregate capital as

$$K(t) \equiv \sum_{i\in\{1,2\}} K_i(t),$$

this implies

$$g_{K,i} = sAK(t)^{-(1-\alpha)} - \delta.$$

Since both sectors grow at the same rate after time T, aggregate capital evolves according to

$$\dot{K}(t) = \sum_{i\in\{1,2\}} g_{K,i}K_i(t) = \left[sAK(t)^{-(1-\alpha)} - \delta\right]K(t),$$

which simplifies to

$$\dot{K}(t) = sAK(t)^\alpha - \delta K(t).$$

state inherits the asymmetries generated during the transition.

The Persistence of Specialization. This toy model delivers a simplified version of the history dependence mechanism in the paper. With a permanent productivity gap, the economy converges to full specialization in the more productive sector. However, even when productivity differences are only temporary, the sectoral composition of capital continues to reflect the history of those differences in the long run. Once technologies become identical again, aggregate capital converges to the standard Solow steady state, but sectoral capital shares remain pinned down by the relative accumulation that occurred during the transition.

The key mechanism is that world prices are fixed. As a result, temporary productivity differences are not offset by relative-price adjustments, allowing them to translate into persistent differences in capital accumulation. In the full model, the relevant state variable is not only sectoral capital, but the wealth-weighted distribution of entrepreneurs within each sector. The full model therefore endogenizes the accumulation mechanism through wealth heterogeneity and financial frictions, allowing the magnitude and persistence of sectoral differences to depend on the evolving distribution of wealth and productivity within sectors.

K Empirical Appendix

This appendix reports additional empirical results supporting the mechanisms emphasized in the paper, but using fixed initial exposure shares instead of dynamically changing ones. It presents local-projection estimates of the dynamic response of sectoral employment shares to external demand shocks, showing persistent sectoral reallocation patterns following favorable external conditions. The appendix also reports firm-level regressions linking capital and labor growth to lagged external demand shocks, firm productivity, and their interaction. Together, these results provide preliminary evidence consistent with the paper's central mechanism, in which external demand conditions interact with heterogeneous productivity and financial frictions to shape accumulation and specialization dynamics over time.

Table K.1: External demand shocks and firm expansion

	Panel A. Capital growth				
	ℓ=1	ℓ=2	ℓ=3	ℓ=4	ℓ=5
Demand shock	0.209	-0.249*	0.084	-0.073	-0.105
	(0.147)	(0.136)	(0.157)	(0.074)	(0.118)
Lagged productivity	0.062***	0.063***	0.062***	0.062***	0.063***
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)
Shock × productivity	-0.026*	0.036**	-0.024	0.013	0.014
	(0.015)	(0.015)	(0.023)	(0.012)	(0.020)
Observations	4,521	4,521	4,521	4,521	4,521
	Panel B. Labor growth				
	ℓ=1	ℓ=2	ℓ=3	ℓ=4	ℓ=5
Demand shock	-1.305***	0.223	-0.346	0.144	-0.105
	(0.316)	(0.590)	(0.422)	(0.557)	(0.226)
Lagged productivity	0.067***	0.063***	0.066***	0.064***	0.065***
	(0.019)	(0.019)	(0.018)	(0.017)	(0.018)
Shock × productivity	0.133***	-0.023	0.075	-0.009	0.005
	(0.037)	(0.078)	(0.055)	(0.085)	(0.032)
Observations	4,532	4,532	4,532	4,532	4,532

Notes: The table reports firm-level regressions of capital and labor growth on lagged sector-level dynamic demand shocks, lagged firm productivity, and their interaction. Panel A uses capital growth as the dependent variable, and Panel B uses labor growth. Columns correspond to shock lags from one to five years. Dynamic demand shocks are constructed from changes in external import demand using initial Peruvian export shares as exposure weights. Lagged productivity is measured using the firm-level productivity residual described in the text. All regressions include sector and year fixed effects. Standard errors are clustered at the sector level. Significance levels are denoted by * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$.

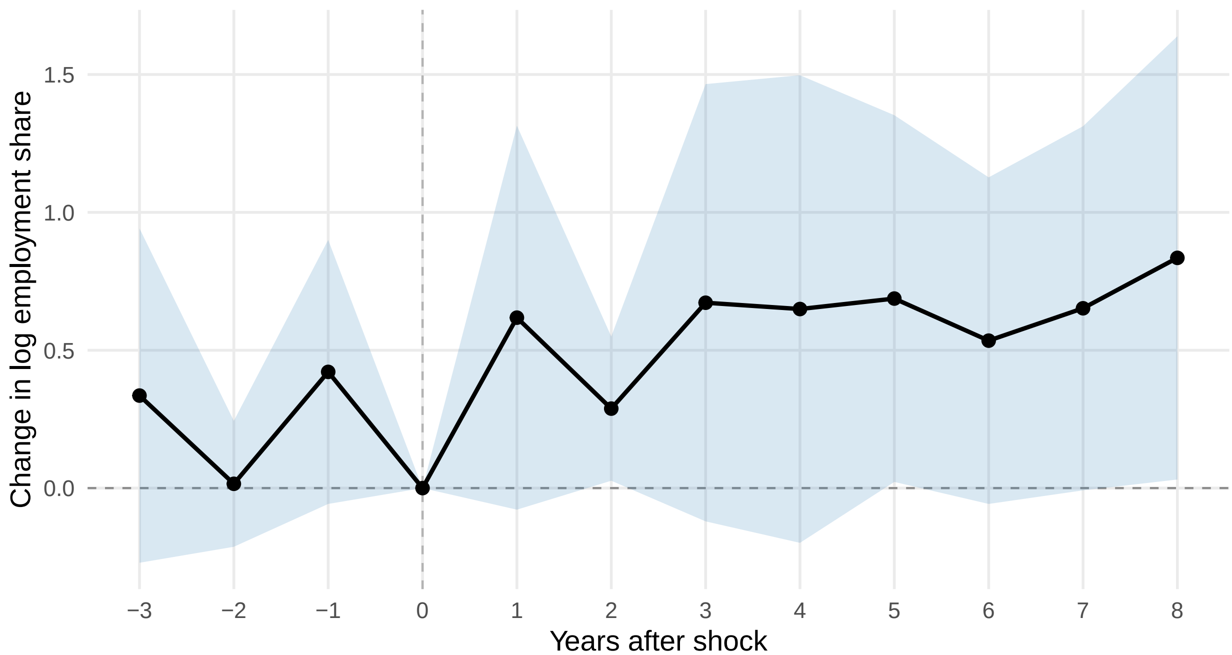
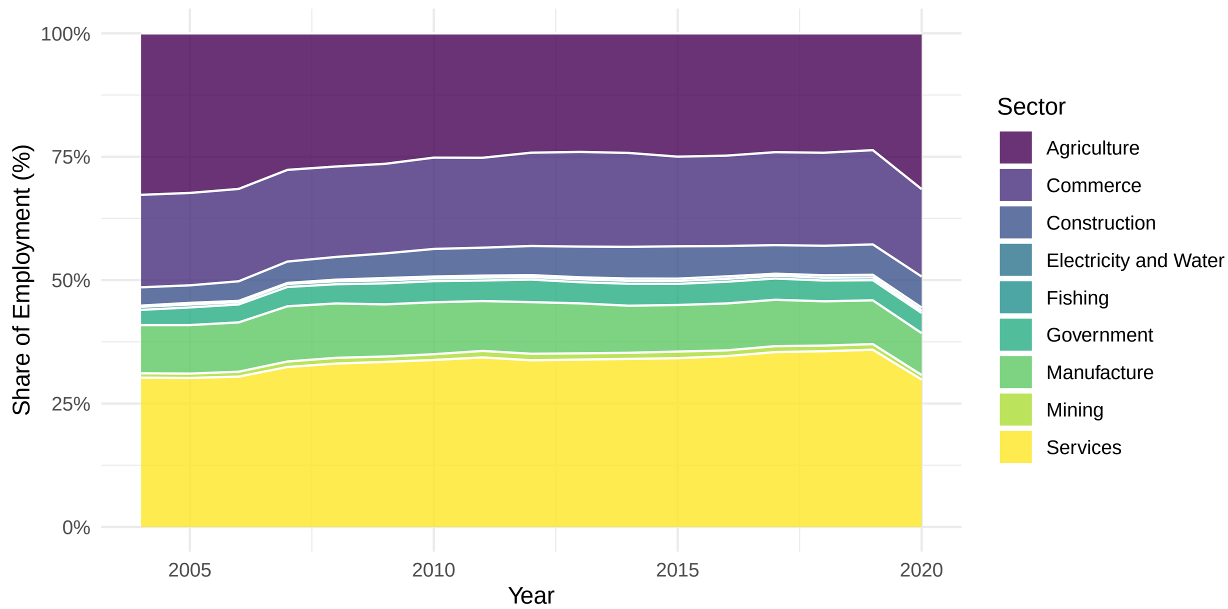
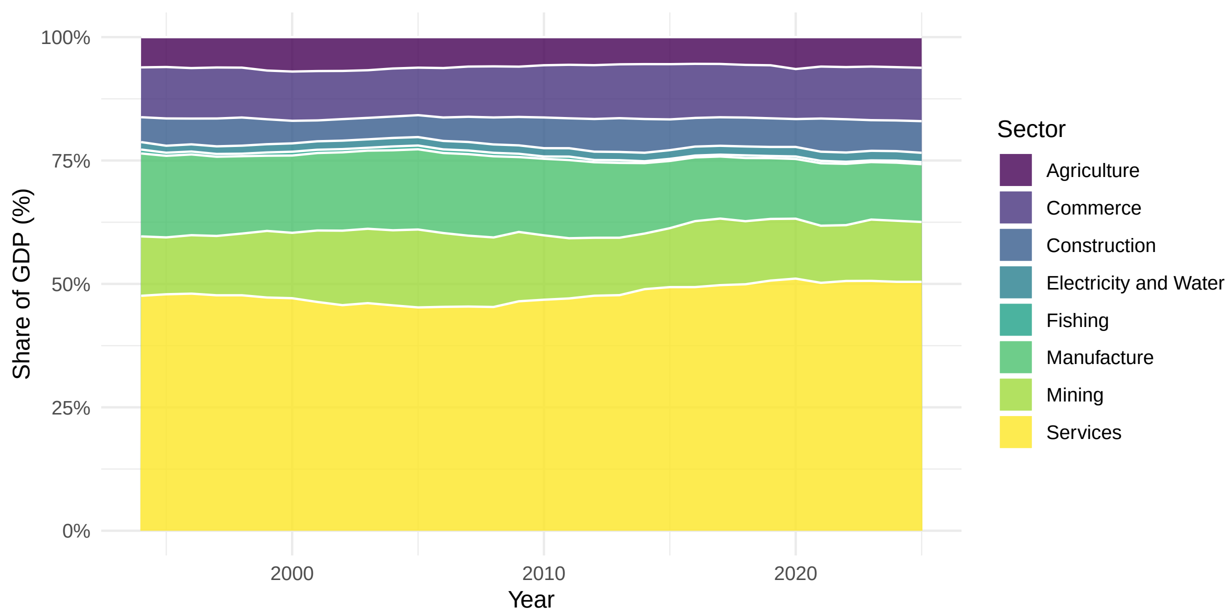


Figure K.1: Dynamic Response of Sectoral Employment Shares to External Demand Shocks

Notes: The figure plots local-projection estimates of the dynamic response of sectoral employment shares to external demand shocks in Peru. The horizontal axis reports years relative to the shock, where negative values correspond to pre-trends and positive values to post-shock responses. The vertical axis reports cumulative changes in log sectoral employment shares. The shock measure is constructed using changes in external import demand weighted by the initial Peruvian export exposure across destination markets and sectors. Shaded bands denote 95 percent confidence intervals based on standard errors clustered at the sector level. Sectoral employment shares are computed using household survey data from ENAHO aggregated to ISIC Rev. 3 two-digit sectors. The dashed vertical line marks the shock period $(h = 0)$, and the dashed horizontal line marks zero change in sectoral employment shares.



(a) Employment



(b) Output

Figure K.2: Sector Composition of Peruvian Economy

Notes: Panel a): Shares represent each sector's percentage of total employment in Peru, ENAHO 2004–2020. Panel b) Shares represent each sector's percentage of GDP, BCRP Series.